Re: Is continuum completely filled up?

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- *From*: Dave Seaman <dseaman@xxxxxxxxxxxx>
- *Date*: Thu, 18 Jan 2007 12:40:51 +0000 (UTC)

On Thu, 18 Jan 2007 10:44:34 GMT, Andy Smith wrote:

> In message <eomd8t$9nu$1@xxxxxxxxxxxxxxxxxxxxxxxxxxx>, Dave Seaman <dseaman@xxxxxxxxxxxx> writes
>
> > > Thank you, understood that, and I can see that things are
> > > inevitably
> > > more tricky than they might seem. But I don't see that that
> > > answers my
> > > simplistic argument about gaps between the numbers?
> >
> > I don't understand what it is that you think has not been answered. You seemed to be struggling for some way to reduce the distance between points to 0. I did three things: (1) I showed you how to reduce the distance to 0 by considering sets of points instead of individual points, (2) I pointed out that reducing the distance to zero is not sufficient to preclude gaps, and (3) I showed you how the concept of completeness addresses the problem of gaps without using distance.
>
> Yes, I understood that.

I don't think you even understood (1), based on your question.

> > If that isn't answering your question, then I evidently have not understood your question. Please ask it again.
>
> Thanks. Virgil pointed out that my use of the word "gap" is inadvisable. My simple train of thought (I can't see how you can ever cover a continuous line with a set of points, however many points you have) went e.g.:

> − take two reals. There is an open interval between them
> − insert 2 more reals between them. There are now 4 numbers, 3 open intervals.
> − iterate N−2 times − you now have 2^N reals and 2^N − 1 open intervals.

Ok, so far.

> − if you have a countably infinite number of bits, then after a countably infinite number of iterations, you still have as many open intervals as distinct real numbers?

That line of reasoning holds only as long as there is such a thing as a "next" point in the set, since an interval lies between a point and its immediate neighbor. There is not "next" point in the rationals, and therefore there are no intervals in the sense that you describe.

And, even though there are no intervals, there are still gaps in the rationals.

> Of course if that was true, then you
> could still continue the process ad infinitum, but that would mean that there are not enough bits to address the reals (and you would still always have as many open intervals as numbers).

No, not always. Only after each finite number of points has been added.

> As I said earlier, I am sure that this line of reasoning is not original and is fallacious, but what is wrong with it?

Properties of finite sets do not necessarily extend to infinite sets.

−−
Dave Seaman
U.S. Court of Appeals to review three issues
concerning case of Mumia Abu−Jamal.
<http://www.mumia2000.org/>
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