

Re: Cantor Confusion

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David Marcus writes

I can see that I am irritating you.

No, you aren't.

It is not deliberate. It is a consequence of a misunderstanding on my part of something fundamental.

I'm sure!

You say, is there such a y ? Well, if we consider the mapping $w = \lim_{n \rightarrow \infty} x^n$, $0 \leq x \leq 1$, we can consider that as the limit of a succession of mappings.

This is simply a language problem. The meaning of what is written is not what you think. It is possible to write what you are thinking: see below.

An analogy would be if I tell you what the word "cat" means and you ask why can't a cat be the animal that barks. It could be, but it isn't. We have another name for that animal.

So, for example we can think of some representative points located in x

Perhaps I should have said in $\{x\}$ $0 \leq x \leq 1$ then.

You can't say "in x ". x is a number. We don't know exactly which, but it is still a number. So, just as you can't say "in 7", you can't say "in

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x . " x " is a name for a number, the number x . Similarly, "7" is a name for the number 7.

e.g $x = 0, 1/4, 1/2, 1$ on $[0, 1]$. After x^2 we get $0, 1/16, 1/4, 1$ and after x^n these points map to $0, 1/(4^n), (1/2^n), 1$. But we still have a continuum of points in $[0, 1]$ after n iterations, and each of these map back to an original location in x . And that has to be true after $n \rightarrow \infty$.

There is no "after $n \rightarrow \infty$ ".

– the line is infinitely elastic. Otherwise you could define a space between two reals in x ?

There is then an inverse mapping from the points in w in $[0, 1]$ back to their original locations in x ? And these would all describe the y that you request?

Name a number y with $|y| < 1$ that will work. You can't.

The following is from Chapter 24 of the book "Calculus" by Spivak.

"Let $\{f_n\}$ be a sequence of functions defined on the set A . Let f be a function which is also defined on A . Then f is called the uniform limit of $\{f_n\}$ on A if for every $\epsilon > 0$ there is some [natural number] N such that for all x in A ,

if $n > N$, then $|f(x) - f_n(x)| < \epsilon$.

"We also say that $\{f_n\}$ converges uniformly to f on A , or that f_n approaches f uniformly on A .

"As a contrast to this definition, if we only know that

$f(x) = \lim_{n \rightarrow \infty} f_n(x)$ for each x in A ,

then we say that $\{f_n\}$ converges pointwise to f on A . Clearly, uniform convergence implies pointwise convergence (but not conversely!)."

Another way to write that $\{f_n\}$ converges uniformly to f on A is

$\lim_{n \rightarrow \infty} \sup_{x \in A} |f_n(x) - f(x)| = 0$.

Let $f_n(x) = x^n$. Let $f(x) = 0$. Then $\{f_n\}$ converges pointwise to f on $[0, 1)$, but not uniformly.

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Thanks for taking the time. Spivak should be here soon ...

My mental problem with the limit of x^n was that viewed as mapping of points I couldn't see how you could split $[0,1]$ into $[0,1)$ and 1 by any sort of limit process – I had a mental image of x^n as a y-axis and x as the x-axis. At $n = 1$ it is a line, then progressively curving close to the x-axis as n increases, but always continuous. Viewed from the y-axis, there always seemed to be an inverse mapping.

But I can see now that this is far too simplistic, and I accept the proof. Ta.

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Andy Smith

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