

Re: Is continuum completely filled up?

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Andy Smith wrote:

OK, another ignorant question.

A real number can be defined by a Cauchy sequence

A real number is not a Cauchy sequence, but rather a real number is an equivalence class of Cauchy sequences.

But if a real number can be represented by a sequence of rationals, why do we need any other numbers than rationals?

To be the least upper bounds and greatest lower bounds of sets of rationals, which allows that there exists things such as the square root of 2.

Or is it that the actually infinite set of a sequence of rationals is logically something other?

A convergent sequence of rationals is a Cauchy sequence. But a real number is not just a Cauchy sequence but rather is an equivalence class of Cauchy sequences. The members of the equivalence class all converge to the same limit; so a real number is that set of Cauchy sequences that converge to that real number (the actual definition, though, is not circular as it would seem from what I just wrote, as you can see from such treatments as Suppes's 'Axiomatic Set Theory').

MoeBlee

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