Re: Is continuum completely filled up?

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- *From*: Andy Smith <Andy@xxxxxxxxxxxxxxxxxxx>
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David Marcus writes

> Actually, that would be "constructing the reals".

yes, noted, sorry.

> > Following a rather chaotic line of thought re the title of this thread,
> > what about space filling curves − covering the plane with a curve is the
> > same issue as covering the line with points.
> >
> > Space filling curves e.g. Hilbert, Peano are fractals; there is a
> > generator function − a suitably line defined in e.g. a square. At each
> > iteration the generator is e.g. reduced in linear scale by a factor 2,
> > replicated by 4 (with some defined rotations for Hilbert) and the scheme
> > is then iterated. As I understand it, the gist of the argument that it
> > is space filling is that, as a function of iteration number n, one can
> > show that one can choose an n such that the distance from any point is
> > less than any eta>0 and that the distance reduces for increasing n.
>
> The argument is more complicated than that. What you wrote just shows
> that we can find a sequence of iterations that come arbitrarily close to
> a given point. However, it turns out that the limit defines a curve and
> this curve completely fills the space.
>
> Well, this is kind of backwards. We can do lots of things. Some of them
> accomplish something and some don't. The real numbers are a specific
> thing. If you want to define/construct something else, it may have
> different properties. Why is that surprising?

All that I meant was that one could e.g. have a situation on a plane such that the plane was totally covered both by points and by holes concurrently (each with a fractal dimension of 2). And, possibly, that one could construct the reals on the line such that any specified real number existed , but that there were still holes (at unspecified locations) not covered by defining points (also at unspecified locations). It is probably nonsense −

but, with the 2D example, if you ask, for a given point, is it white or black in the limit, what is your response?

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Andy Smith
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