

Re: Is continuum completely filled up?

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- *From:* "Randy Poe" <poespam-trap@xxxxxxxxxx>
 - *Date:* 23 Jan 2007 12:59:17 -0800
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Andy Smith wrote:

David Marcus writes

Actually, that would be "constructing the reals".

yes, noted, sorry.

Following a rather chaotic line of thought re the title of this thread,
what about space filling curves – covering the plane with a curve is the same issue as covering the line with points.

Space filling curves e.g. Hilbert, Peano are fractals; there is a generator function – a suitably line defined in e.g. a square. At each iteration the generator is e.g. reduced in linear scale by a factor 2, replicated by 4 (with some defined rotations for Hilbert) and the scheme is then iterated. As I understand it, the gist of the argument that it is space filling is that, as a function of iteration number n , one can show that one can choose an n such that the distance from any point is less than any $\epsilon > 0$ and that the distance reduces for increasing n .

The argument is more complicated than that. What you wrote just shows that we can find a sequence of iterations that come arbitrarily close to a given point. However, it turns out that the limit defines a curve and this curve completely fills the space.

Re: Is continuum completely filled up?

Well, this is kind of backwards. We can do lots of things. Some of them accomplish something and some don't. The real numbers are a specific thing. If you want to define/construct something else, it may have different properties. Why is that surprising?

All that I meant was that one could e.g. have a situation on a plane such that the plane was totally covered both by points and by holes concurrently (each with a fractal dimension of 2). And, possibly, that one could construct the reals on the line such that any specified real number existed, but that there were still holes (at unspecified locations) not covered by defining points (also at unspecified locations). It is probably nonsense – but, with the 2D example, if you ask, for a given point, is it white or black in the limit, what is your response?

I think I understand what you're saying, and it seems to be the same thing that I think Toshiaki is saying. Namely:

1. Construct (define) the reals $[0,1]$.
2. Construct (define) a line segment in some manner not requiring the use of real numbers.
3. Map reals to points on the line segment.
4. Conjecture: bijection between the two sets is impossible, i.e. the cardinality of the line is higher in Cantor's sense than that of numbers in $[0,1]$.

Is it something like that? I'm having trouble even envisioning what step 2 would look like.

– Randy

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