Re: Is continuum completely filled up?

Source: http://sci.tech−archive.net/Archive/sci.math/2007−01/msg05159.html

---

- *From*: "Randy Poe" <poespam−trap@xxxxxxxxx>
- *Date*: 24 Jan 2007 09:05:30 −0800

On Jan 24, 11:17 am, Andy Smith <A...@xxxxxxxxxxxxxxxxxxxx> wrote:

> > Well, this is kind of backwards. We can do
> > lots of things. Some of them
> > accomplish something and some don't. The
> > real numbers are a specific
> > thing. If you want to define/construct
> > something else, it may have
> > different properties. Why is that surprising?
>
> All that I meant was that one could e.g. have a situation on a
> plane
> such that the plane was totally covered both by points and by
> holes
> concurrently (each with a fractal dimension of 2). And,
> possibly, that
> one could construct the reals on the line such that any
> specified real
> number existed , but that there were still holes (at unspecified
> locations) not covered by defining points (also at unspecified
> locations). It is probably nonsense − but, with the 2D
> example, if you
> ask, for a given point, is it white or black in the limit, what is
> your
> response?

I think I understand what you're saying, and it seems to be the same
thing that I think Toshiaki is saying. Namely:

1. Construct (define) the reals [0,1].

2. Construct (define) a line segment in some manner not requiring
the use of real numbers.

3. Map reals to points on the line segment.

4. Conjecture: bijection between the two sets is impossible, i.e.
the cardinality of the line is higher in Cantor's sense than
that of numbers in [0,1].

Is it something like that? I'm having trouble even envisioning
what step 2 would look like.Speaking intuitively, I would have said that the
line in[0,1] is a

1−dimensional space and it is up to somebody to demonstrate that you can
cover it with (an uncountable infinity of) points of zero dimension.

The point of my post was that you can't even ask the
question "do the reals cover it" until "it" is defined. Any
attempt I might make to define the line (for instance
in terms of distance from 0) depends on the real numbers.

So I don't know what "it" is. If I think of it as a representatioin
of [0,1], then of course [0,1] covers it because [0,1] and
"it" are essentially the same thing. "It" has no existence (in
my scheme) outside of [0,1].

Let me be even more specific. I will define "the line segment
from P1 to P2" as the set of points P
{P : P = a*P1 + (1−a)*P2, 0 <= a <= 1}

So everything in this set is defined by a real number a
between 0 and 1. There's no such thing as a point which
is not associated with such an a, since that's my membership
test. Do you see that it's obvious there is no member of
my set not corresponding to some a?

I'm leaving the floor open to other types of definitions, but I
don't know what they are.

You seem to be conceiving of points which have a distance
from one end or the other, but that distance is not a real
number. Is that correct?

− Randy

.