Re: A card game probability

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- *From*: matt271829−news@xxxxxxxxxxx
- *Date*: 26 Jan 2007 17:09:52 −0800

On Jan 26, 9:35 pm, "Faton Berisha" <fberi...@xxxxxxxxxx> wrote:

> On Jan 26, 5:09 pm, "Faton Berisha" <fberi...@xxxxxxxxxx> wrote:
>
>> Now, when I read your message, and again the original question, it seems to me too that what the poster meant was to find the probability of drawing at least once a card with a value (n−1) mod 13 + 1, where n is the number of the trial. It is, of course, a completely different problem.
>
> So let's solve the problem; or equivalently put, we have a deck with cards put faceup arranged in a sequence (e.g., sorted by their suit and rank), and another shuffled deck with cards turned facedown. As we flip the cards from the latter one, we put them sequentially, in the order of appearance, beside the cards from the first one, thus forming pairs of cards from each deck. We want to find the probability of having at least a pair of cards with the same value (i.e. same rank, disregarding the suit).
>
> The probability of flipping a card with the value equal to its pair in the n−th trial, providing that we haven't flipped pairwise equal valued cards in previous n−1 trials, is
>
> bar p_n = Pr(bar A_n | bar A_1 bar A_1 ... bar A_{n−1})
> = sum_{i=0}^4 binom{n−1}i prod_{j=0}^{i−1} (4−j)/(48−j)
> prod_{j=0}^{n−i−2} (44−j)/(48−i−j) (52−n+i−3)/(52−n+1).

[Sorry if this appears more than once. Google is misbehaving.]

I don't quite follow this. I'm guessing that "bar" signifies the complement, so "bar p_n" means the probability of *not* getting the first match at card n. Is that right? So if n = 1 should we have bar p_n = 12/13? I don't see how to get to that. What does binom{n−1}i mean? Initially I thought it might mean "n−1 choose i" but that seems not to work. And what does "A" signify?

> Hence, the probability of flipping at least one such card in n trials is
>
> p_n = 1 − prod_{n=1}^n bar p_n.

This equation does not make a whole lot of sense as written. I'm guessing that you mean something like q_n = 1 − prod_{k=1}^n bar p_k, where q_n is the probability of getting at least one match in n trials. But then why are you multiplying the complementary probabilities and subtracting from one? If p_k is the probability of getting the first match at card k, then the probability of getting at least one match in the first n cards is sum_k=1^n p_k, isn't it? For example, suppose (in a different problem) that p_1 = 1/2 and p_2 = 1/2. We should have q_2 = 1, but your method seems to give an answer of 3/4.

> As it is, the formula is not easily evaluated for large values of n. I made a quick test in my computer, and these are the corresponding values of p_n that I obtained:
>
> (0.07692, 0.14770, 0.21282, 0.27274, 0.32788, 0.37863, 0.42533, 0.46832, 0.50788, 0.54429,
> 0.57781, 0.60866, 0.63705, 0.66319, 0.68724, 0.70937, 0.72973, 0.74846, 0.76568, 0.78152,
> 0.79606, 0.80942, 0.82168, 0.83291, 0.84319, 0.85258, 0.86114, 0.86892, 0.87597, 0.88231,
> 0.88799, 0.89300, 0.89738, 0.90112, 0.90419, 0.90656, 0.90818, 0.90894, 0.90868, 0.90717,
> 0.90401, 0.89858, 0.88982, 0.87578, 0.85269, 0.81230, 0.73394, 0.55369, 0.00001, Indeterminate,
> Indeterminate, Indeterminate)
>
> It is obvious that the accuracy decreases rapidly for values of n near 52. I prescribe that to loss of significant digits due to roundoff error.
>
> Regards,
> Faton Berisha

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