Re: Rational Numbers/Irrational Numbers

Source: http://sci.tech−archive.net/Archive/sci.math/2007−01/msg05863.html

* *From*: Virgil <virgil@xxxxxxxxxxx>
* *Date*: Sat, 27 Jan 2007 16:29:17 −0700

In article <epfl09$7cg$2@xxxxxxxxxxxxxxxxxxxxxxxxxx>,
Dave Seaman <dseaman@xxxxxxxxxxxx> wrote:

> On Sat, 27 Jan 2007 01:28:41 −0500, David T. Ashley wrote:
>
>> <davidmarcus@xxxxxxxxxxxx> wrote in message
>> news:1169874202.106385.326130@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
>>
>>> On Jan 26, 11:36 pm, "David T. Ashley" <d...@xxxxxxxx>
>>> wrote:
>>>
>>>> "Leo" <newsdon...@xxxxxxxxxxx> wrote in
>>>> messagenews:1169780763.086460.114690@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
>>>>
>>>>> Which set has more
>>>>> numbers, the set of rational
>>>>> numbers or the set of
>>>>> irrational numbers?
>>>>
>>>> Well, the set of irrational numbers has at
>>>> least twice as many elements
>>>> as
>>>> the set of rational numbers.
>>>>
>>>> Think about the following functions:
>>>>
>>>> f(x) = PI + x
>>>> g(x) = PI + PI + x
>>>>
>>>> Every rational number x can be paired with
>>>> at least two irrationals.
>>>>
>>>> So, I'm going to go with "irrational" as being
>>>> bigger.
>>>
>>> Right answer. Wrong reason. The rationals are countable.

The
irrationals are uncountable. The rationals have Lebesgue measure zero.
The irrationals in [0,1] have Lebesgue measure 1.

I was just clowning around ... but OK, let's explore your logic.

In order for me to have stated the "wrong" reason, there has to be a counterexample where my test fits but where there are not "more".

Please provide a counterexample.

Consider one of the standard proofs that the rationals are countable. We define a mapping f: N −> Q+ by listing the positive rationals in the following order:

1/1, 1/2, 2/1, 3/1, 2/2, 1/3, 1/4, 2/3, ...

Notice that each rational number appears many times in the list. For example, 1/1, 2/2, 3/3, ... are all the same number, and likewise 1/2, 2/4, 3/6, ...

This obviously proves that there are infinitely many times more natural numbers than there are rationals. Or does it?

Or, instead of surjecting the naturals to the rationals, one can inject the rationals to the naturals:

Given the rational q = m/n, where q = 0 as a rational is 0/1 and otherwise m and n are coprime and n > 0

Define f:Q −−> N : q = m/n |−−> f(q) by
f(0) = 1
if m < 0 then f(q) = (2^(−m))*(3^n)
else f(q) = 5^m * 7^n
.