

Re: A card game probability

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- *From:* matt271829-news@xxxxxxxxxxxx
 - *Date:* 27 Jan 2007 15:30:55 -0800
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On Jan 27, 1:16 pm, "Faton Berisha" <fberi...@xxxxxxxxxxxx> wrote:

On Jan 27, 2:09 am, matt271829-n...@xxxxxxxxxxxx wrote:

I don't quite follow this. I'm guessing that "bar" signifies the complement, so "bar p_n " means the probability of *not* getting the first match at card n . Is that right? Not exactly. Here bar p_n is just a notation. I explained what

precisely it means.

If it troubles you, just denote it by another symbol, say q_n .

So if $n = 1$ should we have bar $p_n = 12/13$? I don't see how to get to that. If you substitute $n=1$, that's exactly what you'll get: $\bar{p}_1 = 12/13$.

(See the computation in my previous message.)

What does $\text{binom}\{n-1\}i$ mean? Initially I thought it might mean " $n-1$ choose i " but that seems not to work. Yes, it means $\{n-1\}$ chose i , and yes it does work.

And what does "A" signify? Denote by A an event. Then \bar{A} is the complement of A .

Denote by $\text{Pr}(A | B)$ the conditional probability of an event A given the occurrence of event B .

Denote by A_n : The card flipped in the n -th trial matches its pair.
Then,

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$$\Pr(\bar{A}_n \mid \bar{A}_1 \bar{A}_2 \dots \bar{A}_{n-1})$$

is the probability of not having a match in the n-th trial, given that no match occurred in previous trials; i.e., the probability of not having a match after n trials given the fact that no match occurred in n-1 first trials.

$p_n = 1 - \prod_{k=1}^n \bar{p}_k$. This equation does not make a whole lot of sense as written. I'm

guessing that you mean something like $q_n = 1 - \prod_{k=1}^n \bar{p}_k$. You are, of course, right here. I mistakenly used the same symbol for

the iterator and the upper limit.

It should be

$$p_i = \prod_{k=1}^i \bar{p}_k$$

for the limit of having at least one match in n trials.

But then why are you multiplying the complementary probabilities and subtracting from one? If p_k is the probability of getting the first match at card k, then the probability of getting at least one match in the first n cards is $\sum_{k=1}^n p_k$, isn't it?

...Be careful here. In my notation \bar{p} and p are not complementary

probabilities (although A and \bar{A} are complementary events).

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Denote by $\Pr(A | B)$ the conditional probability of an event A given the occurrence of event B .

Denote by A_n : The card flipped in the n -th trial matches its pair. Then,

$\Pr(\bar{A}_n | \bar{A}_1 \bar{A}_2 \dots \bar{A}_{n-1})$

is the probability of not having a match in the n -th trial, given that no match occurred in previous trials; i.e., the probability of not having a match after n trials given the fact that no match occurred in $n-1$ first trials.

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[More Google Grief. Sorry if this appears more than once.]

Right, I figured out how you get the value of the expression to be $12/13$ for $n = 1$. I didn't realise that you were assuming $\binom{n-1}{i} = 0$ when $i > n-1$. I just looked at it and thought "that must be wrong".

There are several other things I don't understand in your reply, but rather than labouring over those, let's cut to the chase and look at your final result, which I quote below:

"Hence, the probability of flipping at least one such card in n trials is

$$p_n = 1 - \prod_{i=1}^n \bar{p}_i$$

As it is, the formula is not easily evaluated for large values of n .

I

made a quick test in my computer, and these are the corresponding values of p_n that I obtained:

(0.07692, 0.14770, 0.21282, ..."

I agree with the first result ($n = 1$). For $n = 2$ I calculate the probability as the sum of the following cases:

First card = 1 (prob $4/52$); second card = anything (prob 1)

First card = 2 (prob $4/52$); second card = 2 (prob $3/51$)

First card not 1 or 2 (prob $44/52$); second card = 2 (prob $4/51$)

Total probability = $4/52 * 1 + 4/52 * 3/51 + 44/52 * 4/51 = 98/663 = 0.14781...$

And for $n = 3$, using an analogous method, I get a probability of $3533/16575 = 0.21315...$

These are rather different from your results.

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