

Re: How to solve this natural log problem?

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- *From:* David W. Cantrell <DWcantrell@xxxxxxxxxxx>
 - *Date:* 29 Jan 2007 21:07:11 GMT
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"Dirk Van de moortel" <dirkvandemoortel@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx> wrote:

"David W. Cantrell" <DWcantrell@xxxxxxxxxxx> wrote in message
[news:20070129122419.199\\$HL@xxxxxxxxxxxxxxxxxxx](mailto:news:20070129122419.199$HL@xxxxxxxxxxxxxxxxxxx)

"Dirk Van de moortel"
<dirkvandemoortel@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx>

wrote: <amy_burton2007@xxxxxxxxxxx> wrote in message
news:1170055954.369782.189430@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

How to solve for x?

$$4000x = -6.9 - \ln x$$

Take excel and write a number in cell A1.
In cell A2 enter the formula
 $= A1 - (4000 * A1 + 6.9 + \text{LN}(A1)) / (4000 + 1/A1)$
and copy this down to about A20.

This is Newton's method with $f(x) = 4000 x + 6.9 + \ln(x)$

You're going to need a very small number to start with in A1
Try putting 1, 0.1, 0.01, 0.001 etc... in A1 and see what happens.

If you don't have excel, just take a high precision calculator
and you'll find the result in a few minutes.

My first preference would be to use the Lambert W function, as Rob did.

Yes, although excel doesn't have the Lambert W function

And I don't have Excel. :-)

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(and I have *never* seen a home/garden/kitchen calculator offer it), there is, at least on this group, a strange tendency to mention Lambert W whenever someone is looking for a solution to this kind of equations.
Strange :-)

My second preference would be to use some analysis to get an approximate solution. Then, if that's not sufficiently accurate, it at least gives a good value for A1 in Newton's (or some other iterative) method.

Knowing that x must be a small positive number, the equation

$$a x = -b - \ln(x)$$

has the approximate solution

$$x = (2a + 3c - \text{Sqrt}(-2a^2 + 24ac + 9c^2)) / (a(a - 2c))$$

where $c = e^b$.

Using $a = 4000$ and $b = 6.9$ in the above, we get the approximate solution

$x = 0.000308...$ For comparison, the solution, as mentioned by Rob, is

$$x = 0.0003016...$$

Yes, that's what Excel gives me after a mere 5 iterations with a seed of 0.001.

OK, let's assume that Amy is using a common calculator. If she heeds your advice to "Try putting 1, 0.1, 0.01, 0.001 etc... in A1 and see what happens.", she will be frustrated by an error message at the second iteration using 1, 0.1 or 0.01 for A1. (The error occurs because her calculator cannot handle the log of a negative number.)

I'm advocating that one not pick A1 "blindly". If you don't like the approximation I gave earlier (because it's too complicated?), then merely use

$$1/(a + e^b)$$

a much cruder approximation, for A1. Doing so will still avoid any error messages.

I had given the more accurate approximation earlier because I thought that

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perhaps it would be accurate enough for Amy's purpose without any iterations at all. (Her "6.9" might be an approximation with just two significant digits.)

David

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