

# Re: coefficients

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*Source:* <http://sci.tech--archive.net/Archive/sci.math/2007-02/msg02888.html>

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- *From:* The World Wide Wade <[waderameyxiii@xxxxxxxxxxxxxxxxxxxxxxxx](mailto:waderameyxiii@xxxxxxxxxxxxxxxxxxxxxxxx)>
  - *Date:* Wed, 14 Feb 2007 16:43:49 -0800
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In article <20070214.084753@xxxxxxx>, rob@xxxxxxxxxxxxxxxx (Rob Johnson) wrote:

In article <1171470176.060012.170740@xx>, se16@xxxxxxxxxxxxxxxx wrote:

On 14 Feb, 16:17, r...@xxxxxxxxxxxxxxxx (Rob Johnson) wrote:

In article <1171464223.131431.98...@xx>,

"Toxician" <fridgechemis...@xxxxxxxxxx> wrote:

$$F(x) = (x^2 + x^3 + x^4 + x^5)(x + x^2 + \dots + x^7)(1 + x + \dots + x^{15})$$

What is the coefficient of  $x^{15}$  ? I try to multiply all of them but it becomes too long.

$$\begin{aligned}
 F(x) = & x^3 + 3x^4 + 6x^5 + 10x^6 + 14x^7 + 18x^8 + 22x^9 \\
 & + 25x^{10} \\
 & + 27x^{11} + 28x^{12} + 28x^{13} + 28x^{14} + 28x^{15} + 28x^{16} \\
 & + 28x^{17} \\
 & + 28x^{18} + 27x^{19} + 25x^{20} + 22x^{21} + 18x^{22} + 14x^{23} \\
 & + 10x^{24} \\
 & + 6x^{25} + 3x^{26} + x^{27}
 \end{aligned}$$

So the answer is 28.

Rob Johnson <r...@xxxxxxxxxxxxxxxx>  
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## Re: coefficients

Indeed, but since only  $x^{15}$  is needed you could just look at  $0+0+0+1+2+3+4+4+4+4+3+2+1+0+0+0$  (and just slide left and right for the other sums)

Yes, I did resort to the easy way and used program to multiply the polynomials. However, to explain your sum to the OP, we first multiply the first two polynomials, which is pretty simple:

$$x^3 + 2x^4 + 3x^5 + 4x^6 + 4x^7 + 4x^8 + 4x^9 + 3x^{10} + 2x^{11} + x^{12}$$

Then, to compute the coefficient of  $x^{15}$ , we match the terms in this product with those of  $(1+x+\dots+x^{15})$  that make  $x^{15}$ . We start with the  $x^3$  term in  $(1+x+\dots+x^{15})$  and end with the  $x^{12}$  term:

$$1 + 2 + 3 + 4 + 4 + 4 + 4 + 3 + 2 + 1 = 28.$$

As others have pointed out, it's easier than that. Multiplying the first two polys together with no simplification yields the sum of  $4*7 = 28$  terms, each of the form  $x^k$ ,  $k = 3, \dots, 12$ . Each of those terms can be paired with exactly one term in the last polynomial to give  $x^{15}$ . The coefficient of  $x^{15}$  is therefore 28.

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