Re: Complex Analysis − polar form notation!?!?!?!

Source: http://sci.tech−archive.net/Archive/sci.math/2007−02/msg04729.html

- *From*: Narcoleptic Insomniac <i_have_narcoleptic_insomnia@xxxxxxxxx>
- *Date*: Sun, 25 Feb 2007 17:39:58 EST

On Feb 25, 2007 2:49 PM CT, i.love.jeevitha@xxxxxxxxx wrote:

> On Feb 24, 7:57 pm, Narcoleptic Insomniac
> <i_have_narcoleptic_insom...@xxxxxxxxx> wrote:
>
> > On Feb 24, 2007 2:56 PM CT, i.love.jeevi...@xxxxxxxxx wrote:
> >
> > > > My complex analysis book says...
> > > >
> > > > (u,v,x,y in |R)
> > > > Suppose w = u + i*v is the value of the complex
> > > > function " f " at z = x + i*y
> > > >
> > > > i.e., w = u + i*v = f(x + i*y)
> > > >
> > > > Each of the real numbers u and v depends on the
> > > > real variables x and y, and it follows that f(z)
> > > > can be expressed in terms of a pair of real−valued
> > > > functions of two real variables x, y:
> > > >
> > > > w = f(z) = u(x,y) + i*v(x,y)
> > > >
> > > > ***
> > > >
> > > > I stared at this for a while, dazed and a little
> > > > confused by the notation, then I fell back on
> > > > the "complex number is just an ordered pair of
> > > > reals" idea. So I thought of this last equation as
> > > > just something like:
> > > >
> > > > (u,v) = f(x, y) = (u(x,y) , v(x,y))
> > > >
> > > > Then it made sense to me. But after this the book

> says...
>
> ***
>
> If the polar coordinates r and @ (theta) are used instead of x and y, then
>
> u + i*v = f(r*exp(i*@)) where w = u + i*v and z = r*exp(i*@). In that case we may write
>
> f(z) = u(r, @) + i*v(r, @)
>
> ***
> Other than laughing at the use of "w" for no reason whatsoever, I was a bit confused by what this notation really meant (I had a feeling but I couldn't pinpoint it).
>
> In the x,y coordinate case, since z= (x,y), nothing special was being done by splitting the equation up into two real valued parts. But here I don't know what's going on.
>
> I think the polar form case is drastically different than the x,y case. We're essentially using a composite function here, no?
> z = r*exp(i*@) would be an intermediate function mapping r, @ coordinates to x,y coordinates.

No, there need not be such an intermediate mapping. In fact, I believe your troubles are comming from this very notion.

> But throughout the book so far the author treats polar form as something very natural, as if it's just "substitute r*exp(i*@) for z and everything works." No mention of composite functions.

Why do you think it is so unnatural?

We're not 'substituting' r e^{i@} for z − we are just using the polar coordinate system instead of the cartesian system.

Perhaps you were mislead by the statement that z = x + iy. Usually z is just taken to mean any arbitrary complex number − the properties of the field of complex numbers are independent of the coordinate

>> system we choose.
>>
>>> Can anyone make sense of this f(z) = u(r, @) + i*v(r, @) ?
>>>
>>> The expression makes some sense but what is it really? We have a function of the variable z, which really is a (x,y) equal to a two functions of (r, @), but not just that, one of them is multiplied by i.
>>>
>>> So this translates to (since w = (u,v) and z = (x,y)):
>>>
>>> (u,v) = f(x,y) = (u(r, @) , v(r, @)) ?
>>>
>>> This make no sense whatsoever to me. Someone please help.
>>
>> Actually, the more I read your post, the more confused I get!
>>
>> What you have is a mapping f:C −−> C. Instead of using cartesian coordinates in the domain, we are going to use polar coordinates.
>>
>> Each point z in the domain can be represented by pair (r, @) by letting z = r e^{i @}. There is no need to begin with (x, y) and somehow obtain (r, @) − we are *starting* with (r, @).

> When we write z = r e^{i @}
>
> We are mapping (r,@) to (x,y) because
>
> z = r e^{i @} = r(cos@ + i sin@) = rcos@ + i rsin@ = (rcos@, rsin@)

No, when we write z = r e^{i @} we are attaching the ordered pair (r, @) to the element z in the complex plane.

In other words, we are interpreting the field of complex numbers as the set of ordered pairs

C = {(r, @) : r >= 0, 0 <= @ < 2pi}

...that satisfy certian algebraic properties.

> So z = r e^{i @} is like an intermediate function
> transforming coordinates (r,@) to (x,y) , correct?

No, but the mapping f:C −−> C with

f(r, @) = f(r e^{i@}) = (r cos(@), r sin(@))

...that you gave above is.

I believe someone earlier suggested that you experiment with the mapping g:C −−> C such that g(z) = z^2; let's consider that for a moment. We are going to use polar coordinates in our domain and cartesian in our codomain.

This means that we are going to BEGIN with z = r e^{i@} − WE'RE ASSOCIATING EVERY ELEMENT z WITH SOME ORDERED PAIR (r, @) − and we're going to associate every element f(z) = u + iv in the image with the ordered pair (u, v).

Now g(z) = z^2 = r^2 e^{2i@} = r^2 [cos(2@) + i sin(2@)]

...so...

g(r, @) = (r^2 cos(2@), r^2 sin(2@))

...which implies...

g(r, @) = u(r, @) + i v(r, @)

...where...

u(r, @) = r^2 cos(2@) ...and... v(r, @) = r^2 sin(2@).

There are two important things that should be noted here.

First, there was no need for an intermediate mapping f(r, @) = r cos(@) + i r sin(@); we simply use the identity e^{it} = cos(t) + i sin(t).

Sure, we *could* consider this identity as a mapping on it's own right (and it's a very important one), but there is no *need* to. For if we did, the composition would look something like this.

(g o f)(z) = (g o f)(r, @) = (g o f)(r e^{i@})
= g(r cos(@), r sin(@))

= g(r cos(@) + i r sin(@))
= [r cos(@) + i r sin(@)]^2
= r^2 cos(@)^2 + 2i r cos(@) sin(@) − r^2 sin(@)^2.

The other thing that should be noted is the geometric interpretation of the mapping z |−−> z^2.

Recall that we had g(r, @) = r^2 e^{2i@}. If we consider the image in polar coordinates instead of cartesian we get g(r, @) = (r^2, 2@). This implies that the mapping will square the length of a vector and double the argument.

Regards,
Kyle Czarnecki

> In your explanation of
>
> f(z) = f(r e^{i @}) = u(r,@) + i v(r,@)
>
> Even the form of the value is a complex number of form u + i v
>
> say (x,y) = g(r,@) = (rcos@, rsin@) −same as− r e^{i@}
>
> *ignore the actual details of g(r,@) for now, just assume a function exists that can do this mapping*
>
> and our function is f(x,y) = (h(x,y) , k(x,y))
>
> so,
>
> (u,v) = f(g(r,@)) = (h(g(r,@)) , k(g(r,@)))
>
> so u = h(g(r,@))
> v = k(g(r,@))
>
> So it is in fact an intermediate function z=g(r,@) at work. Am I right?

(see above)
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