

## Re: Analytic Functions in 3D?--

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- *From:* JEMebius <jemebius@xxxxxxxxxx>
  - *Date:* Tue, 27 Feb 2007 23:44:40 +0100
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I post this a second time because of a frustrating typing error and because I forgot to remove \_no spam\_. I typed "Well, there is on 3D field over the reals." instead of "Well, there is no 3D field over the reals."

Jeff wrote:

>Hello  
>  
>I've been studying the remarkable properties of analytic functions in the  
>complex plane and their relationship to conformal mapping.  
>  
>I was wondering if there was any extension of this to 3 dimensions – though  
>I suspect not.  
>And what about higher dimensions? Will the quaternions do for 4D what  
>complex numbers do for 2?  
>  
>Any pointers or keywords for searching would be appreciated.  
>  
>Many Thanks  
>Jeff.  
>

I am afraid you are right to guess that there is no extension to 3D –

The ingredients for differentiability are the four principal arithmetical operations and the limit concept, in other words: one needs a topological field, preferably commutative, the topology preferably derived from a metric, the field being complete as a metric space.

Well, there is no 3D field over the reals.

As regards quaternions: the norm in the quaternion skew field yields the well-known Euclidean metric with its accompanying topology. Multiplication is not commutative, so one needs to distinguish between left-division ( $p^{-1}q$ ) and right-division ( $qp^{-1}$ ), and consequently, between left- and right-difference and differential quotients.

I am not aware of a fully developed quaternion differential calculus. Google and Wikipedia might be helpful.

Concerning conformal mappings of 4D regions:

First look at the 2D case. In the plane we have at every point a combined translation, rotation and magnification, plus something infinitesimal.

## Re: Analytic Functions in 3D?---

Something similar happens in 4D: 4D translation, 4D rotation, magnification, plus something infinitesimal.

The snag is in the 4D rotations. They come in several different kinds. Read more about 4D rotations in Wikipedia > SO(4) and in my arXiv articles at <http://arxiv.org/abs/math.GM/0501249> and <http://arxiv.org/abs/math.GM/0701759>,

all of this also reachable from my web page on 4D rotations at <http://www.xs4all.nl/~jemebius/4drot.htm>.

Also read about hyper-Kähler manifolds, a class of 4D Riemann manifolds with special, quaternion-based properties.

Cheers: Johan E. Mebius

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