

# ZFC++

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Hi all,

ZFC+Proper classes+Ur elements'ZFC++':is the set of sentences entailed (from first order logic with identity)by these axioms:

1) Extensionality:  $\forall x \forall y (x=y \leftrightarrow \forall z (z \in x \leftrightarrow z \in y))$ .

2) ur-element:  $\exists x (\forall y (y \in x \leftrightarrow y=x))$ .

Definition 1)  $x$  is a ur-element  $\leftrightarrow \forall y (y \in x \leftrightarrow y=x)$ .

3) Regularity:

$\forall x ((\exists z (z \in x)) \rightarrow \exists y (y \in x \ \& \ \sim \exists c (c \in y \ \& \ c \in x \ \& \ \sim c=y)))$ .

4) Schema of Global comprehension: if  $P$  is a formula in which  $x$  is not free then all closures of:

$\exists x \forall y (y \in x \leftrightarrow (P(y) \ \& \ \exists z (z \in y)))$

are axioms.

Definition 2)  $x=V \leftrightarrow \forall y (y \in x \leftrightarrow (y=y \ \& \ \exists z (z \in y)))$ .

Definition 3)  $x$  is a proper class  $\leftrightarrow \sim \exists y (x \in y)$ .

Accordingly  $V$  is a proper class.

Definition 4)  $x=0 \leftrightarrow \forall y (\sim y \in x)$ .

5) Pairing:  $\forall a \forall b \exists x \forall y (y \in x \leftrightarrow (y=a \ \text{or} \ y=b))$ .

6) Union:  $\forall a \exists x \forall y ((y \in x \leftrightarrow \exists z (z \in a \ \& \ y \in z)) \ \& \ (\exists m (a \in m) \leftrightarrow \exists n (x \in n)))$ .

7) Infinity:  $\exists N \forall 0 \in N \ \& \ (\forall x (x \in N \rightarrow \exists U \{x\} \in N))$ .

with 'U' and {x} having the usual definitions.

8) limitation of size:

$\text{Ax}((\text{Ez}(\text{xez})) \leftrightarrow x \text{ is subnumerous to } V).$

$x \text{ is subnumerous to } V \leftrightarrow \text{Af}((f:x \rightarrow V) \rightarrow (f \text{ is injective} \ \& \ \sim f \text{ is surjective})).$

9) Power:  $\text{AaExAyeV}((\text{yex} \leftrightarrow \text{Az}(\text{zey} \rightarrow \text{zea})) \ \& \ (\text{Em}(\text{aem}) \leftrightarrow \text{En}(\text{xen}))).$

Definition 5)  $x = K \leftrightarrow \text{Ay}(\text{yex} \leftrightarrow y \text{ is a ur-element})$

so  $K$  has all ur-elements as its members.

10) ur-multiplicity:

$\text{Ax}((\text{Az}(\text{zex} \rightarrow \text{zeK}) \ \& \ \text{Eu}(\text{xeu})) \rightarrow \text{Ey}(y \text{ is a ur-element} \ \& \ \sim \text{yex})).$

Accordingly  $K$  is a proper class.

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