Introduction of Viete factorization.

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Factorize a^2 + 2ab + b^2.

Since there are two letters(a, b),
let p(x) = x^2 − (a+b)x + ab.

Since p(a) = p(b) = 0,
a^2 − (a+b)a + ab = 0 ==> a^2 + ab = (a+b)a
b^2 − (a+b)b + ab = 0 ==> b^2 + ab = (a+b)b

so, a^2 + 2ab + b^2 = (a+b)a + (a+b)b
= (a+b)(a+b)
= (a+b)^2

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Factorize a^2 + b^2 + c^2 + 2ab + 2bc + 2ca.

Since there are three letters(a,b,c),
let p(x) = x^3 − (a+b+c)x^2 + (ab+bc+ca)x − abc.

Since p(a) = p(b) = p(c) = 0,
a^3 − (a+b+c)a^2 + (ab+bc+ca)a − abc = 0.
==> a^2 − (a+b+c)a + (ab+bc+ca) − bc = 0 (divide by a)
==> a^2 = (a+b+c)a − (ab+ca)

b^3 − (a+b+c)b^2 + (ab+bc+ca)b − abc = 0.
==> b^2 = (a+b+c)b − (ab+bc) similarly.

c^3 − (a+b+c)c^2 + (ab+bc+ca)c − abc = 0.
==> c^2 = (a+b+c)c − (ca+bc) similarly.

sum...a^2 + b^2 + c^2 = (a+b+c)(a+b+c) − 2(ab+bc+ca)
==> a^2 + b^2 + c^2 + 2(ab+bc+ca) = (a+b+c)(a+b+c).

Maybe, we can apply to various case...

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