Re: Review of Mueckenheims book.

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- *From*: "William Hughes" <wpihughes@xxxxxxxxxx>
- *Date*: 2 Mar 2007 14:23:48 −0800

On Mar 2, 3:58 pm, mueck...@xxxxxxxxxxxxxxxxx wrote:

> On 2 Mrz., 19:31, "William Hughes" <wpihug...@xxxxxxxxxx> wrote:
>
> > Recall: We construct the union tree from finite trees. Finite trees
> > have finitely many nodes. Finitely many nodes are not capable of
> > forming infinitely many paths.
>
> M: There are only a countable number of nodes.
> How can we form an uncountable number of infinite paths

Big snip here.

> Can't you understand or won't you understand?
>
> A finite number of nodes cannot be used to construct an infinite
> number of different paths.
>
> Yes O No O

Yes

> All finite trees have finite numbers of nodes

Yes, (you don't seem to be able to take yes for an answer)

> which for the union of
> finite trees do not add but are the number of the larger tree.

No. If there is a largest tree then it makes sense to say
they "do not add" However, there is no largest finite tree.

> Yes O No O
>
> Therefore the number of nodes is finite even for the infinite union of
> finite trees.

No.

It is trivial to see that given a finite number of nodes there is
a single finite tree that contains all the nodes.
So if the infinite union of finite trees contained a finite number of
nodes, there would be a single finite tree, call it L1, such that L1
contains the infinite union of finite trees.

> Yes O No O
>
> This is because the natural numbers are finite, even if we consider
> infinitely many of them.

Yes, but despite the fact that each natural number is finite there
are infinitely many of them

> Yes O No O
>
> What is difficult about that?

Nothing except for the statement that the
number of nodes in a union of finite trees "do not add".
This only makes sense if there is a largest tree
in the union. There is no largest finite tree, so there

is no largest tree in the union of all finite trees.

> H: The union tree is constructed from an infinite number of finite trees. The union tree contains an infinite number of nodes.

> The complete tree may contain an infinite number of nodes. The union tree cannot contain an infinite number of nodes, because this would mean the existence of an infinite natural number.

i: Each finite tree contains a finite number of nodes

ii: The union tree only contains finite trees.

iii: The union tree contains a finite number of nodes.

iii does not follow from i and ii. Saying that the union tree contains an infinite number of nodes does not mean that a finite tree must contain an infinite number of nodes.

> Do not mistake: There are infinitely many trees in the union, but every tree has a finite number of nodes. And in the union of two or more trees there is not the sum of nodes of the trees but only the number of nodes of the largest tree of the union.

If there is a largest tree of the union this is true. However, there is no largest finite tree, so there is no largest tree in the union of all finite trees.

> This is a valid approach to construct the union tree.

> Yes, and you end up with an infinite number of nodes.

> Not in the union of finite trees. You end up with an infinite number of trees. But the number of nodes in that union is finite.

No. You end up with the set consisting of every node that is in at least one finite tree. This is an infinite set.

> > OK the infinite path p(oo) consists of the union of
> > all finite paths having only nodes with value zero.
>
> > So it is said. As the nodes of the finite paths must be taken from
> > finite trees, their number is finite though not bounded.
>
> > i: each finite path has a number of nodes which
> > is finite but not bounded
>
> For each finite path he number of nodes is finite and bounded.
>
> For the union of finite paths the number of nodes is finite but not bounded.
>
> > ii: p(oo) is the union of finite paths
>
> > iii: p(oo) has a number of nodes which is finite but not bounded.
>
> That has to be figured out.

More snipping

> Since no one claimed this there is no problem. What was claimed
> is that an infinite number of paths, each of which has a finite number
> of nodes with value zero, can yield a single infinite path with an
> infinite number of nodes with value zero.

Maybe. But there is no infinite number of finite paths.

In other words "What I said was nonsense, however, rather
than appologizing, I will make yet another claim."

The new claim

i: every finite path is finite

ii: every set of finite paths contains only finite paths

iii: there are a finite number of finite paths

No, iii still does not follow from i and ii.

> I hope you will not switch to the claim now that there were
> natural
> numbers of infinite size because there are infinitely many of
> them?

> No. But there are infinitely many of them, so the *set* of
> natural numbers is of infinite size.

Correct. But completely irrelevant in this context.

> iii does not follow from i and ii. Therefore saying that the *set* of
> natural

> numbers is of infinite size does not mean that there is a natural
> number of infinite size.

Correct.

See below

> But a finite number of nodes cannot supply an infinite number
> of paths.

> Your claim is that *any* union of trees is a tree.

Yes, but all are finite.

> Let T be the union of all finite trees.

More snipping

> iii does not follow from i and ii. T is the union of an
> infinite number of finite trees and therefore has an infinite number
> of nodes.

> Wrong. This would mean the infinite union of natural numbers contains
> an infinite number.

No.
saying that the *set* of natural
numbers is of infinite size does not mean that there is a natural
number of infinite size.

Here the number of nodes is the size of the *set* of natural numbers.
Saying that there is an infinite number of nodes does not mean that
there is an infinite natural number.

> Remember: The union of finite trees is a finite tree, namely the largest one in that union.

There is no largest finite tree so the union of all finite trees is not the largest one in the union.

− William Hughes

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