Re: Why is the Russell Paradox necessary?

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- *From*: "Butch Malahide" <fred.galvin@xxxxxxxxx>
- *Date*: 5 Mar 2007 20:15:31 −0800

On Mar 5, 9:31 pm, "Calvin" <cri...@xxxxxxxxxxxxxx> wrote:

> In ordinary set theory, in which Russell's Paradox is used
> to show that assuming the existence of a set of all sets
> leads to that paradox, ie. a contradiction, I wonder why
> the Russell Paradox method is necessary.

Did someone tell you that it is necessary?

> We know that the cardinality of the set of all subsets of a
> given set is greater than the cardinality of the given set.
> But if we assume the existence of a set of all sets, by
> definition it contains all of its subsets. Therefore the
> cardinality of the set of all subsets of the set of all sets
> cannot be greater than the cardinality of the set of all
> sets, which is a violation of what we know about
> cardinality. Hence there is no set of all sets.

This is another way of proving the nonexistence of a universal set. Of *course*, if there are two different ways to do the same thing, then neither one of them is "necessary", as we could get by with the other one. It is not at all unusual to have alternative proofs of the same theorem, e.g., the hundreds of different proofs of the theorem of Pythagoras about right triangles. In the case you're talking about, the two proofs are not so very different. As I understand it (could be wrong, this is hearsay), it came about this way. The proof via cardinalities−−if there is a universal set V, then P(V) = V, contradicting Cantor's theorem that |P(X)| > |X| −− came first. Russell studied that proof, and *simplified* it to the one−line argument known as "Russell's paradox". Namely, if P(V) = V, then the identity function f(x) = x is a bijection from V to P(V). The inequality |P(V)| > |V| is proved by constructing, for a given function f:V−−>V, the "diagonal" set D = {x in V: x is not in f(x)}. Applying the diagonal construction to the identity function f(x) = x, we get the so−called Russell set.

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