Re: Review of Mueckenheims book.

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- *From*: Virgil <virgil@xxxxxxxxxxx>
- *Date*: Tue, 06 Mar 2007 21:02:21 −0700

In article <1173230899.320093.5150@xxxxxxxxxxxxxxxxxxxxxxxxxxxx>,
"MoeBlee" <jazzmobe@xxxxxxxxxxx> wrote:

> On Mar 6, 4:55 pm, David Marcus <DavidMar...@xxxxxxxxxxxxxx> wrote:
>
>> MoeBlee wrote:
>>
>>> On Mar 6, 3:07 pm, David Marcus
>>> <DavidMar...@xxxxxxxxxxxxxx> wrote:
>>>
>>>> The vast majority of math textbooks define
>>>> functions as maps between two
>>>> specified sets.
>>>
>>> I don't know about the vast majority. I've seen different kinds
>>> of
>>> definitions in different books. Plenty of textbooks in abstract
>>> algebra, analysis, and topology DO give the usual set
>>> theoretic
>>> definition of 'function', while other textbooks in those
>>> subjects (I
>>> don't know the comparative proportion) give definitions such
>>> as you
>>> mentioned.
>>
>> Topology by Munkres: "A function f is a rule of assignment r, together
>> with a set B that contains the image set of r. ..."
>
> That's one for you.
>
> But this is silly. I can also bring out books in abstract algebra,
> analysis, and topology, that use the set theoretic definition of a
> function.

I have no idea which is the majority definition. And I HOPE I can refrain from playing the silly game of hauling out a bunch of definitions from different books.

It's easy enough to see that some books use the set theoretic definition and others don't. Belaboring that with lists of examples and quotes is indeed tedious.

> Real Analysis by Royden: "By a function f from (on on) a set X to (or into) a set Y we mean a rule ..."

That is aside the set theoretic definition insofar as it mentions a rule rather than a set of ordered pairs. But notice just that a definition is given of 'a function from X into Y' (or variously, 'to' or 'onto') (which is to describe a 3−place relation among the function f and the sets X and Y) does not contradict that we may also give a definition of 'is a function' alone (which is a 1−place predicate).

> Algebra by Mac Lane and Birkhoff: "A function f on a set S to a set T assigns ..."

Again, granted, 'assigns' is different from the set theoretical definitions. But again, as per remarks above regarding S and T.

> Naive Set Theory by Halmos: "If X and Y are sets, a function from (or on) X to (or into) Y is a relation f such that ..."

Again, and especially here, there is no conflict between this and the set theoretic definition. I have said ALL ALONG that we can define 'is a function from X to Y' as well as just plain 'is a function'.

> A Mathematical Introduction to Logic by Enderton: "A function is a relation F with the property that for each x in dom F there is only one y such that <x,y> in F."

That is equivalent to the set theoretic definition.

> Topics in Algebra by Herstein: "If S and T are nonempty sets, then a mapping from S to T is a subset, M, of S X T such that ..."

The word 'function' is not in that. Though, granting 'mapping' in place of 'function', I've already granted that a many mathematics books make such defintions.

WHAT IS YOUR POINT in hauling out a bunch of definitions when I'VE ALREADY GRANTED that many mathematics textbooks don't present the usual set theoretic definition? A statistical comparison? Sheesh, how tedious can we get! You'll find plenty that have the usual set theoretic definition and plenty that don't. So what?!

But now that we're here, let me say what I don't like about the above definition. It gives MORE information than it needs to. All the DEFINITION needs to say is:

A function from S to T is a relation such that....

THEN it is entailed that that relation is a subset of SxT. That entailed part doesn't have to be part of the definition. Now, of course, I recognize that the author achieves economy and packs information in by the way he gives the definition, which is fine with me. I'm just saying that for an appreciation of the very fine points as to distinguishinb between what is a bare bones definition and what is entailed from that definition, I prefer not to give the kind Herstein does with that one. And in set theory, we would prove the existence of that function by taking a subset of SxT, but the DEFINITION of 'function from S to T' does not have to mention SxT since the definition is not itself the existence proof.

> Real Analysis and Probability by Dudley: "Informally, given sets D and E, a function f on D is defined by assigning to each x in D one (and only one!) member f(x) of E. Formally, a function is defined as a set f of ordered pairs <x,y> such that for any x, y, and z, if <x,y> in f and <x,z> in f, then y = z."
>
> Dudley is a bit on the fence, but I'll give him to you. Perhaps not a large sample, but 5/7 of the books require a function to have a specified codomain.

No, five of the seven require mentioning a specfic codomain to define 'f is a function from X to Y'. And *I* require that too! How many times do I have to say it:

Defining 'is a function' is different from defining 'is a function from X to Y'. For the former, no specific codomain needs to be mentioned, but for the latter a specific codomain is mentioned. I've always said that; never denied it.

Do you see ANYTHING about what I'm saying?

And do you see that we can find textbooks that give all kinds of

> definitions, and that while many will not be the usual set theoretic definition, plenty of them will be, which is what I've said all along.
>
> Now I hope we can move on, since I really don't know what possible disagreement there is here, UNLESS it's as to some statistical comparison, which I think is just a silly waste of our time, since I've not even made any statistical claim other than that plenty of books go one way and plenty go another way, which is apparent enough.
>
> MoeBlee

The issue here is whether there is a standard definition of function, that is widely accepted and widely propagated over a fairly wide variety of mathematical fields, including analysis, which requires that a function have a specified codomain.

In fact, outside of Calculus, I am not aware of any area of mathematics in which a specific codomain for each function, mapping or operator is not required.

That it is not always required in calculus and elementary analysis I am aware, but even there it sometimes is.

I will grant Moblee freedom to continue to use "function" in the sense he prefers so long as he will grant me the same.
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