Re: Review of Mueckenheims book.

Source: http://sci.tech−archive.net/Archive/sci.math/2007−03/msg03987.html

- *From*: cbrown@xxxxxxxxxxxxxxxxx
- *Date*: 18 Mar 2007 12:58:03 −0700

On Mar 17, 4:57 pm, Tony Orlow <t...@xxxxxxxxxxxxx> wrote:

> cbr...@xxxxxxxxxxxxxxxxx wrote:
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> > On Mar 17, 8:59 am, Tony Orlow <t...@xxxxxxxxxxxxx> wrote:
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> > > cbr...@xxxxxxxxxxxxxxxxx wrote:
> > >
> > > > On Mar 13, 9:22 am, Tony Orlow
> > > > <t...@xxxxxxxxxxxxx> wrote:
> > > >
> > > > > cbr...@xxxxxxxxxxxxxxxxx
> > > > > wrote:
> > > > >
> > > > > > On Mar 12,
> > > > > > 2:11 pm,
> > > > > > Tony Orlow
> > > > > > <t...@xxxxxxxxxxxxx>
> > > > > > wrote:
> > > > > >
> > > > > > > Where
> > > > > > > we
> > > > > > > can
> > > > > > > count
> > > > > > > in
> > > > > > > sequence
> > > > > > > from
> > > > > > > one
> > > > > > > element
> > > > > > > to
> > > > > > > any
> > > > > > > other,
> > > > > > > that
> > > > > > > neighborhood
> > > > > > > is
> > > > > > > finite,
> > > > > > > even

if unbounded. Where we can never count between some pair of objects, such as between, say, ...1111 and ....2222, they are actually infinitely distant elements of a sequence, since successor() exists.

Ermm...
what th'!?!
Cheers −
Chas

Was that a question?

One question might be "what is an example of a neighborhood that is finite yet unbounded?"

The atmosphere. :)

In what sense is the atmosphere finite? In what sense is it unbounded? If the atmosphere is finite, then there is a molecule of the atmosphere which is the farthest from Earth. A distance greater than that distance is a bound on the atmosphere, no?

No comment?

> If you add 0 forever, you will never get anywhere, right? If
> you append
> points to points, you can never make a line of any length,
> right? But,
> you have lines of finite length, and they have an uncountable
> number of
> points within them. No countable number of points can
> constitute a line
> segment of any length, but an uncountable number can, in
> theory.
>
> Consider this an analogy to the addition of finite units. No
> countable
> number can achieve any infinite measure, where such a thing
> is properly
> established. Aleph_0 is, the way I see it, the equivalent of the
> smallest positive number, on the infinite scale. The smallest
> positive
> number does not exist, finite or infinite.
>
> So, "countably infinite" means "finite but unbounded" to me.

So in your terminology, the distinction between something which is finite and something which is not finite is so vague that a mathematical object can simultaneously have the property of being finite and also /fail/ to have the property of being finite.

No comment?

That argues for your perhaps developing a new term that corresponds to what is usually meant by "finite", so that the term actually /describes/ the property of being "finite" in the usual sense. Otherwise, I have no idea what you mean by "finite". In the usual sense, it is not possible for a mathematical object to be both finite and not finite.

> > > In a countable set, there are
> > > only a finite number
> > > of elements between any
> > > two specific elements.
> >
> > Counterexample: The set of rationals in [0,1]
> > is a countable set, and
> > there are an infinite number of elements
> > between any two distinct
> > elements of that set.
>
> Not in the order in which they are countable.

Is 1/2 between 1/4 and 3/4 "in the order in which they are countable"?

Starting at 1/1 and traversing the standard table diagonally to make a sequence, no, 1/2 is reached before 1/4, which is reached before 3/4:

1/1 1/2 1/3 1/4 ...

2/1 2/2 2/3 2/4 ...

3/1 3/2 3/3 3/4 ...

4/1 4/2 4/3 4/4 ...

5/1 5/2 5/3 5/4 ...

You cannot state two rational numbers included in Cantor's diagonal proof of their countability which are infinitely distant from each other in that sequence.

Yes that is /an/ order in which 1/2 is not between 1/4 and 3/4. You claimed that it was /the/ order in which they are are countable; and no such /unique/ order exists.

> Why don't you give me two
> rational numbers, which are not finitely distant in the
> pseudo−sequential non−quantitative (read, bogus) ordering
> of the

>>> rationals through sparse diagonalization? Which comes
>>> infinitely beyond
>>> any other, in that "countable" sequence? Hmmm....

>> So you yourself claim that your statement only makes sense if we use a "bogus" ordering of the rationals? Hmmm indeed!

> That's the ordering used by Cantor to prove their countability. What I said was that no two elements in any countable set are infinitely distant from each other in any linear ordering that makes them countable.

Actually, what you said was:

>>>>> In a countable set, there are
>>>>> only a finite number
>>>>> of elements between any
>>>>> two specific elements.

And that was insufficient to make sense out of your statement.

If what you /intended/ to say was:

If a set S is countable, then there is a total order < of S such that for all distinct x, y in S with x < y, there are at most a finite number of elements z such that x < y < z.

then I would of course agree.

> If this sequential order is valid for the rationals, then this fact applies. If you have a problem with the fact that there are an infinite number of rationals between any two in their natural quantitative order...

Why would I have "a problem" with that statement? I'm the one who mentioned it.

In fact one can equally say:

If a set S is countable, then there is a total order < of S such that for all distinct x, y in S with x < y there are an infinite number of elements z such x < z < y.

But that isn't the same as saying: In a countable set, there are an infinite number of elements between any two specific elements.

> , then you should consider the possibility that non−quantitative orderings of subsets of the reals are not suitable for relative measure of those subsets.

Erm? What th'?!?

> There are an infinite number of adic numbers between ...111 and ...222, no? And the adics each have a distinct successor, yes? What was the question?

> Another question might be "are you aware of the difference between the definitions of a list of elements from a set, a total order on the elements of that set, and a well−ordering of the elements of that set?"
> Cheers − Chas

> Make it relevant to the topic, and we'll discuss that.

> You said:

> "Where we can never count between some pair of objects, such as between, say, ...1111 and ...2222, they are actually infinitely distant elements of a sequence, since successor() exists."

> By "since successor() exists", you seem to imply that:

>> "/Because/ successor() exists, they are actually infinitely distant elements of a sequence".

> Since successor(x) is defined for every such string, it constitutes a sequence.

Are you aware of the definition of the term "sequence"? It appears not.

> Since there exist elements more than any finite number of successions from each other, it is uncountable.

Let succ(x) = x + 2. Using this definition, is not succ() a successor function on the naturals, with two elements (0 and 1) having no predecessor? What "number of successions" is it from 0 to 1? Is N therefore uncountable? I.e., is N is a set which has the property of being countable, and which also /fails/ to have the property of being countable?

>> But the existence of a successor function does /not/ imply a set is (or can be made into) a sequence or list, in the usual definition. It doesn't even imply a set is infinite, in the usual definition. Nor does it necessarily impose a total order; it may be at best a partial order.

> If xeS −> s(x)eS, and ExeS AyeS s(y)<>x, then you have a countably infinite sequence at the very least.

S = {0,1,2}; s(0) = 1, s(1) = 2, s(2) = 1 is a finite set satisfying the above premises.

But you get 10 points for actually producing a clear yet false statement, as opposed to gibberish! Seriously, that is progress!

> The question is whether one can
> have an uncountable sequence, which the adics clearly are.

No, the question is "what do you mean by an uncountable sequence, of which you assert the adics are an example?" A sequence (by any definition I know) is countable, because it is always basically equivalent to some function f : N −> S, and N is countable. You obviously don't mean "an uncountable sequence is a sequence (which is countable by definition) which is not countable", because that would be silly. But what /do/ you mean?

You /might/ mean "suppose C is an uncountable set with a total order, then an uncountable sequence on S is a function f : C −> S", but there are /many/ different types of sets which are uncountable, and have a total order, but whose order types are "as different" as the order types of N and Q.

> > Let T be the set of all triangles in the plane. If t is a triangle, define successor(t) to be t after being rotated around the origin by pi/sqrt(2) radians. Every t has a unique successor. Every triangle t has a unique triangle for which t is its successor.
>
> Indeed, and it constitutes an uncountably long sequence which is actually circular.

Erm? What th?'?

> > Despite the existence of successor(), T is not a sequence; nor can it be made into a sequence, because T is uncountable
>
> Uncountable sequences aren't a problem for me.

Well, "uncountable sequences" may or may not be "a problem" for anyone else either; of course, depending on what the heck you're talking about.

> The only thing that
> distinguishes this from a sequence for me is the fact that every point is a successor as well as having one. But, like the integers, where this

may also be considered true, it's not surprising that an overall
circular nature may manifest itself.

What is "circular" about the ordering of integers?

Yes, poetically speaking, a circle has no beginning or end; and the integers have no begininng or end. Also, my love for my niece has no beginning or end. Is that all you mean to say?

Also, we can certainly define "t1 < t2" as "there is a natural number n such that t2 is the result of applying the successor function n times to t1". But there are triangles t and u such that it is not the case that t < u or u < t or t = u.

Right. Some will be uncountably many successions beyond any other.

No.

Firstly, and most obviously, there is /no/ number of applications of rotating an equilateral triangle by /any/ amount which will yield a non−equilateral triangle. So we cannot say (purely from the ordering imposed by succ() on T) that an equilateral triangle is <, >, or = a triangle which is not equilateral.

Secondly, because sqrt(2) is irrational, there is no number of successive applications of succ() to a triangle t such that we arrive at the triangle t rotated by pi/(10^(10^1000)) radians. So these two triangles t, and t rotated by pi/(10^(10^1000)), while in some sense being "very, very close" to being "the same" by "distance", are still incomparable as regards my given definition of <, >, = as derived from succ().

You
can define '<' in this way, or you can define "t1 < t2" as "t2−t1>0".

Erm? What th'?!? We can?

Let t1 be the unit equilateral triangle with 1 vertex at the origin and base on the x−axis. Let t2 be the right triangle with sides 1,1,sqrt(2)/2 having its right angle at the origin. Is t1 − t2 > 0? Or is t1 − t2 <= 0? How are we to know for arbitrary triangles s, t, except by directly asking you?

And how does the ordering you claim we can define relate to your claim that "/because/ there is a successor function, blah blah..."? The question isn't, "/can/ we somehow totally order T?", because that is fairly obviously true given AofChoice for any set (and thus trivial). The question is "is there a specific total ordering on T, /since/ succ() exists?"

> If
> t2 is infinitely past t1, then t1<t2. Like I said, the problem in this case is the circularity of the uncountable system.

If t1 is some equilateral triangle, and t2 is t1 rotated about the origin by pi radians, then is t1 "infinitely past" t2, or is it the other way round? t1 is not a member of the sequence (t2, succ(t2), succ(succ(t2)), ...); nor is t2 a member of the sequence (t1, succ(t1), succ(succ(t1)), ...). So how are we to know, except by directly asking you?

Again, the question isn't "/can/ we somehow totally order T?", because the answer is trivially "yes". The question is how do you suggest we /define/ this total order using /only/ succ(), when you aren't around to help us?

>> These observations arise from the definitions of "sequence" and "total order"; which is why I asked: ""are you aware of the difference between the definitions of a list of elements from a set, a total order on the elements of that set, and a well−ordering of the elements of that set?"
>>
>> Cheers − Chas

> So, what do you call what I am calling an uncountable sequence? A nonexistent concept?

No, I would instead call it a poorly−defined concept.

Given your fixation on "succ()", the best I can understand what you are thrashing about for by "uncountable sequence" is a function which is indexed by an ordinal A whose /ordinality/ is greater than omega. By extension of the example that f : N −> S is a sequence of S, you want "uncountable sequence" to be a function f : A −> S, where A is an ordinal and omega in A (i.e., omega < A where < is defined by set inclusion); and that gives us a function which is "like" a sequence, only "longer".

But there are ordinals greater than omega in the /ordinality/ sense (e.g., the ordinal succ(succ(omega)) > omega) which have the same / cardinality/ as omega; and that is where you begin getting mixed up. Such ordinals are /not/ uncountable, they are /countable/. There are indeed uncountable ordinals; but countable/uncountable does /not/ refer to comparison by /ordinality/, rather by /cardinality/.

This becomes further confused when you then mix in other order types, such as the orderings of Q and R and the adics, which are not orders which can be compared with ordinals by their /ordinality/ because Q, R and the adics are /not ordinals/ (nor are they order−isomporphic to an ordinal).

And your concept of "succ()" is vague enough that it ends up including order types where succ() does not even impose a total order, as in the examples S and T I gave above (which are pre−orders and partial orders, resp.).

Thus my question, which I will expand slightly: "are you aware there is a difference between the definitions of a sequence, a cardinal, an ordinal, a well order, a total order, a partial order, a pre−order?".

Every sequence is indexed by a cardinal (omega). Every cardinal is an ordinal. Every ordinal imposes a well order. Every well−order is a total order. Every total order is a partial order. Every partial order is a pre−order.

But:

Not every pre−order is a partial order. Not every partial order is a total order. Not every total order is a well order. Not every well−order is order−isomporphic to an ordinal. Not every ordinal is a cardinal. Not every cardinal is the index of a sequence: only omega fits that bill.

Cheers − Chas

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