Re: The Collatz discrete primes!

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On Mar 23, 8:01 am, "Danny" <fasttrac...@xxxxxxxxxxxxx> wrote:

> > > Just spotted this In OEIS −−− A127781
> > > "The Hailstone pure numbers"
> > > So my special list of primes could be called
> > > the Hailstone pure primes!
>
> > That's Pure Hailstone primes −−− A127928.
>
> I missed that one, I guess I just reinvented
> the wheel!

Great minds think alike, eh?

> What interest me more about all the pure hailstone
> numbers is the difference pattern they leave.

At first I didn't realize you had gone back to _all_
pure numbers. Changing my program to show all numbers
(not just primes or odds) I get

(0) 405 [0]
(1) 406 [0]
(2) 407 [0]
(0) 408 [0]
(1) 409 [2]
(2) 410 [0]
(0) 411 [0]
(1) 412 [0]
(2) 413 [0]
(0) 414 [0]
(1) 415 [0]

(2) 416 [0]
(0) 417 [0]
(1) 418 [0]
(2) 419 [2]
(0) 420 [0]
(1) 421 [2]
(2) 422 [0]
(0) 423 [0]
(1) 424 [0]
(2) 425 [0]
(0) 426 [0]
(1) 427 [0]
(2) 428 [0]
(0) 429 [0]
(1) 430 [0]
(2) 431 [2]
(0) 432 [0]
(1) 433 [2]
(2) 434 [0]
(0) 435 [0]
(1) 436 [0]
(2) 437 [0]
(0) 438 [0]

where for (m) n [p]

− m is modulo 3
− n is the number
− p is prime flag (0=composite,2−prime)

pure on the left, impure to the right.

And when I calculate differences, I get something like (not the same sequence)

1 2 3
1 2 3 3 3
1 2 3
1 2 3 3 3
1 2 3
1 2 3 3 3
1 2 3
1 2 3 3 3
1 2 3
1 2 3 3 3 3 3
1 2 3 3 3 3 3
1 2 3 3 3
1 2 3
1 2 3 3 3
1 2 3
1 2 3 3 3

1 2 3
1 2 3 3 3 3 3
1 2 3 3 3
1 2 3
1 2 3 3 3
1 2 3
1 2 3 3 3
1 2

So I think I'm finally on the same page.

> I belive we discussed this on an earlier post
> but delving into it more I discovered this −−
>
> After the initial start these are the only difference
> patterns in the hailstone pure numbers.
>
> 1,2,3
> 1,2,3,3,3
> 1,2,3,3,3,3,3

Which I concur on.

> There never is a sequential repeat of the first
> two patterns (1,2,3) and (1,2,3,3,3) but
> (1,2,3,3,3,3,3) at times will repeat once.
>
> These repeats occur between hailstone pure @
> 405 − 438
> 891 − 924
> 1863 − 1896
> 3159 − 3192
> 3321 − 3354
> 4779 − 4812
> 5265 − 5298
> 6237 − 6270
> 6705 − 6738
> −− etc.
>
> Most of the time the general pattern will be −−
> [123][12333][123][12333][123][1233333][12333][123]−−−
> and once in awhile the repeat of [1233333][1233333]
> but never [123][123] or [12333][12333].
> I wonder why that is?

Ah, the $64 question: why?

There are always rules. Rules have implications. Sometimes the interaction of these implications is hard to see when they are all run together. I may not have crossed every i and dotted every t, but I think it works something like this:

First, every 0(mod3) number is pure and since natural numbers are always in modulo 3 sequence, we start out with

0(mod3) pure
1(mod3) ?
2(mod3) ?
0(mod3) pure

That means no difference can be greater than 3. If that were the end of the story we would have only [3333333...] as a sequence of differences. There is, of course, more to it than that.

We also know that every 2(mod3) is impure (because its sub−branch is only one step above). This applies to both even and odd 2(mod3) numbers. So far, we've got

0(mod3) pure
1(mod3) ?
2(mod3) impure
0(mod3) pure

Which doesn't change our differences. What does change them is whether or not the 1(mod3) is pure or impure (and it can be either). That gives us two possibilities

0(mod3) pure
1(mod3) impure
2(mod3) impure
0(mod3) pure

or

0(mod3) pure
1(mod3) pure
2(mod3) impure
0(mod3) pure

For the first case, we still get a 3, but for the second, we get a 12 instead of a 3 for differences (essentially, a 3 can be replaced by a 12).

So everywhere we see a 12 in the sequence, a 1(mod3) is pure. Wherever you see a 3, the 1(mod3) inside that block is impure.

Next, every _even_ 1(mod3) is impure. Any 1(mod3) is evenly divisible by 3 when you subtract 1, but, by the implication that 3n+1 always results in an even number, you cannot reverse that operation when the 1(mod3) is odd. You can always reverse it when 1(mod3) is even, thus, every even 1(mod3) has a smaller ancestor and thus, every even 1(mod3) is impure.

What does that imply to our sequence? That you cannot have 2 pure 1(mod3) numbers in a row (because at least one will be even). So a pure 1(mod3) implies

0(mod3) pure
1(mod3) pure
2(mod3) impure
0(mod3) pure
1(mod3) impure
2(mod3) impure
0(mod3) pure

and that means every 12 is followed by a 3, so [123] is the smallest possible unit and the sum of a unit must be a multiple of 6

[123] = 6
[12333] = 12
[1233333] = 18

But wait...there's more.

To have two [123] blocks in a row implies that the next numbers are 12..., in other words

[123][123][12...]

which means we have 3 odd 1(mod3) pure numbers in a row but that won't happen since to be pure, the first sub−branch can't be a 2(mod3) and out of three consecutive odd 1(mod3) numbers, at least one will have a 2(mod3) sub−branch. That gives us the major implication

− there cannot be 3 consecutive odd 1(mod3) pure numbers

Of course, you can skip one, but if you do, you get two 3's, the one you skipped and the next one because the 1(mod3) number would be even. So if you don't get [123], you must get at least [12333].

Likewise, the other major implication is that out of 3 odd 1(mod3) numbers, at least one is pure, so we have

− there cannot be 3 consecutive odd 1(mod3) impure numbers which means we can't have a unit larger than [1233333].

Let's see, I haven't covered why you don't see [12333][12333], but my brain is kinda fried right now, I'm sure there's a similar reason.

The answers are always there if you dig deep enough.

Dan

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