Re: question regarding diofantine equations

Source: http://sci.tech−archive.net/Archive/sci.math/2007−04/msg01490.html

- *From*: rob@xxxxxxxxxxxxxx (Rob Johnson)
- *Date*: Wed, 11 Apr 2007 19:09:27 GMT

In article <gerry−A5AC0B.14285711042007@xxxxxxxxxxxxxxxxxx>,
Gerry Myerson <gerry@xxxxxxxxxxxxxxxxxxxxxxxxx> wrote:

> In article <20070410.175730@xxxxxxxx>, rob@xxxxxxxxxxxxxx (Rob Johnson)
> wrote:
>
>> In article <gerry−B2C6C7.08533011042007@xxxxxxxxxxxxxxxxxx>,
>> Gerry Myerson <gerry@xxxxxxxxxxxxxxxxxxxxxxxxx> wrote:
>>
>>> In article <20070410.142106@xxxxxxxx>,
>>> rob@xxxxxxxxxxxxxx (Rob Johnson)
>>> wrote:
>>>
>>>> In article
>>>> <1176237012.557078.287790@xxxxxxxxxxxxxxxxxxxxxxxxxx>,
>>>> "laura" <laura.brandusan@xxxxxxxxx>
>>>> wrote:
>>>>
>>>>> I want to solve diofantine
>>>>> equations of form:
>>>>>
>>>>> ax+by=c,
>>>>>
>>>>> where a, b and c are real
>>>>> numbers and
>>>>>
>>>>> x and y are natural numbers
>>>>> (>=0).
>>>>>
>>>>> Are there any methods for
>>>>> solving this ? I don't want to
>>>>> enumerate all
>>>>> possible pairs (x,y) and to
>>>>> check which ones are good.
>>>>>
>>>>> Or, is there possible to
>>>>> decide if the equation has

solutions without
solving it?

The algorithm is called the extended euclidean algorithm, and one implementation is the Euclid−Wallis Algorithm:

<http://www.whim.org/nebula/math/euclid−wallis.html>

OP wants a, b, and c to be real numbers. If b = 1 and c = 0 then the question of whether a x + b y = c has solutions is the question of whether a is rational. It's going to take a heck of
an extension of Euclid's algorithm to decide whether, say, gamma
is rational.

Yes, I missed that a, b, and c were not necessarily integers. With this generality, there are a couple of cases:

1. { ax + by : x,y are integers } has a smallest positive element, d (equivalently, a/b is rational).

In this case, a/d and b/d are relatively prime integers, and the equation has a solution if and only if c/d is an integer. Solutions are given by the extended euclidean algorithm.

2. { ax + by : x,y are integers } does not have a smallest positive element (equivalently, a/b is not rational).

Solutions here would require a canvassing of all values of x and y.

Although you could get lucky. E.g., pi x − y = sqrt 2 clearly has no solutions in natural x, y − no canvassing required. pi x − y = pi − 1 clearly has the solution x = y = 1. pi x + y = e clearly has no solutions in natural x, y. pi x − y = e is, I think, unknown.

Certainly, if c is given in the form of a solution, the search is very short. Of course, even checking one possible answer, unless c is in such a form, might be quite difficult. Situations can also be easily vacated where one of a, b, and c is transcendental and the others are algebraic, or one is irrational and the others are rational. But in general, this is difficult.

However, I am curious why you claim that pi x + y = e has no solution

in natural x and y, but that it is unknown whether pi x − y = e has a
solution. It would seem that one has a solution if and only if the
other does.

Rob Johnson <rob@xxxxxxxxxxxxxx>
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