Re: teminology

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- *From*: "Zdislav V. Kovarik" <kovarik@xxxxxxxxxxx>
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On Fri, 13 Apr 2007, fishfry wrote:

> In article <1176455217.528539.209410@xxxxxxxxxxxxxxxxxxxxxxxxxxxx>,
> "gius" <giu.giorgi@xxxxxxxxx> wrote:
>
>> Hi, everyone,
>>
>> In mathematics,
>> what does it mean for a map defined in a certain set,
>> say A,
>> to extend "naturally" to a set B which contains as a proper subset
>> the set A ?
>
> It means the "obvious" extension. It's the extension 10 out of 10
> mathematicians would pick if you asked them. In other words there's not
> always a precise definition, but it's obvious.
>
> Example: f:N−>N given by f(n) = n, where N is the naturals. How would
> you extend that "naturally" to the reals? Obviously you'd pick the
> identity function on the reals; even though there are uncountably many
> other extensions that take the same values on N.
>
> Does that help?

A different situation: suppose that the set B has a structure (ordering,
group, topology) and A can inherit some or all of the structure in such a
way that a map defined on A has an extension which is, perhaps with some
added properties, unique.
Example: B can be an inner product space, A a dense linear subspace, and
f is a continuous linear map on A. Then f has exactly one extension which
is linear and continuous on all of B − the natural extension.
Or: B as before, A a closed subspace, f linear continuous on A, then

the "minimal" extension, defined as zero on the orthocomplement of A,
and extended by linearity, would be voted "natural".

Cheers, ZVK(Slavek).
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