Re: Probably haven't seen this one, but...

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- *From*: junoexpress <MTBrenneman@xxxxxxxxx>
- *Date*: 4 May 2007 08:18:43 −0700

On May 3, 1:12 pm, Robert Israel
<isr...@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx> wrote:

> junoexpress <MTBrenne...@xxxxxxxxx> writes:
>
> > Hi,
> >
> > I'm doing some work with a function that is the ratios of (normalized)
> > sinc functions, having the form:
> > f:x−> Sinc(Mx)/Sinc(x) for M a natural number and |x| < 1
> >
> > I'm having to work out some properties of this function, which are not
> > that bad, but in the process, I keep wondering if this ratio has been
> > analyzed before (in other words, I hate to present a lot of detailed
> > derivations only to have someone else say, "Oh yeah, that's just the
> > Gluckenheimer function" and if it has been looked at before, maybe I
> > could get some deeper insight into the solution also.)
> >
> > So I come to the gurus. Is this a function which anyone has seen
> > analyzed before? To my knowledge, it is not in Abrahamowitz and
> > Stegan, and the closest I can come to pinning it on anything known is
> > to say that it's the ratio of 2 spherical Bessel functions (which
> > doesn't seem like a function that's probably been analyzed).
>
> Since sinc(x) = sin(x)/x (for x <> 0), your function is just
> f(x) = sin(Mx)/(M sin(x)). This can also be written as
> U_{M−1}(cos(x))/M where U_k is the k'th Chebyshev polynomial of
> the second kind.
> −−
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This is a very nice observation. There is one problem I am having in applying them to my problem. I need to know (or have a decent bounds) on the absolute value of the first extreme value of U_m(x) after x=0. So in the case of U_2(x), you could solve for the critical points, take the critical point closest to zero (which is not equal to zero) x*, and then compute |U2(x*)|. But of course for M>4, this strategy will not work, and there is no obvious way I can see to get a (close) upper bound to this first extreme value. (Could always do this computationally, but if there was a proven bound, that of course would be better).

Matt

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