Re: Probably haven't seen this one, but...

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- *From*: Robert Israel <israel@xxxxxxxxxxx>
- *Date*: 4 May 2007 09:15:21 −0700

On May 4, 8:18 am, junoexpress <MTBrenne...@xxxxxxxxx> wrote:

> On May 3, 1:12 pm, Robert Israel
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> <isr...@xxxxxxxxxxxxxxxxxxxxxxxxxxxxx> wrote:
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>> junoexpress <MTBrenne...@xxxxxxxxx> writes:
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>>> Hi,
>>>
>>> I'm doing some work with a function that is the ratios of
>>> (normalized)
>>> sinc functions, having the form:
>>> f:x−> Sinc(Mx)/Sinc(x) for M a natural number and |x| < 1
>>>
>>> I'm having to work out some properties of this function,
>>> which are not
>>> that bad, but in the process, I keep wondering if this ratio has
>>> been
>>> analyzed before (in other words, I hate to present a lot of
>>> detailed
>>> derivations only to have someone else say, "Oh yeah, that's
>>> just the
>>> Gluckenheimer function" and if it has been looked at before,
>>> maybe I
>>> could get some deeper insight into the solution also.)
>>>
>>> So I come to the gurus. Is this a function which anyone has
>>> seen
>>> analyzed before? To my knowledge, it is not in
>>> Abrahamowitz and

> > > Stegan, and the closest I can come to pinning it on anything
> > > known is
> > > to say that it's the ratio of 2 spherical Bessel functions (which
> > > doesn't seem like a function that's probably been analyzed).

> > Since sinc(x) = sin(x)/x (for x <> 0), your function is just
> > f(x) = sin(Mx)/(M sin(x)). This can also be written as
> > U_{M−1}(cos(x))/M where U_k is the k'th Chebyshev polynomial of
> > the second kind.
> > −−
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> This is a very nice observation. There is one problem I am having in
> applying them to my problem. I need to know (or have a decent bounds)
> on the absolute value of the first extreme value of U_m(x) after x=0.
> So in the case of U_2(x), you could solve for the critical points,
> take the critical point closest to zero (which is not equal to zero)
> x*, and then compute |U2(x*)|. But of course for M>4, this strategy
> will not work, and there is no obvious way I can see to get a (close)
> upper bound to this first extreme value. (Could always do this
> computationally, but if there was a proven bound, that of course would
> be better).

|sin(Mx)/sin(x)| <= |1/sin(x)|

The peaks will be quite close to points where sin(Mx) = (+/−) 1,
especially
when M is large. It should be possible to get good explicit bounds.

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