Re: Probably haven't seen this one, but...

Source: http://sci.tech−archive.net/Archive/sci.math/2007−05/msg00632.html

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- *From*: Robert Israel <israel@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx>
- *Date*: Fri, 04 May 2007 18:23:33 −0500

David W. Cantrell <DWCantrell@xxxxxxxxxxx> writes:

> [This is to supersede my previous (incorrect) posting.]
>
> Robert Israel <israel@xxxxxxxxxxx> wrote:
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>> On May 4, 8:18 am, junoexpress <MTBrenne...@xxxxxxxxx> wrote:
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>>> On May 3, 1:12 pm, Robert Israel
>>> <isr...@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx> wrote:
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>>>> junoexpress <MTBrenne...@xxxxxxxxx>
>>>> writes:
>>>>
>>>>> Hi,
>>>>>
>>>>> I'm doing some work with a
>>>>> function that is the ratios of
>>>>> (normalized) sinc functions,
>>>>> having the form:
>>>>> f:x−> Sinc(Mx)/Sinc(x) for
>>>>> M a natural number and |x|
>>>>> < 1
>>>>>
>>>>> I'm having to work out some
>>>>> properties of this function,
>>>>> which are
>>>>> not that bad, but in the
>>>>> process, I keep wondering if
>>>>> this ratio
>>>>> has been analyzed before (in
>>>>> other words, I hate to
>>>>> present a lot
>>>>> of detailed derivations only

to have someone else say,
"Oh yeah,
that's just the
Gluckenheimer function"
and if it has been looked
at before, maybe I could get
some deeper insight into the
solution
also.)

So I come to the gurus. Is
this a function which
anyone has seen
analyzed before? To my
knowledge, it is not in
Abrahamowitz and
Stegan, and the closest I can
come to pinning it on
anything known
is to say that it's the ratio of
2 spherical Bessel functions
(which doesn't seem like a
function that's probably
been analyzed).

Since sinc(x) = sin(x)/x (for x <> 0), your
function is just
f(x) = sin(Mx)/(M sin(x)). This can also be
written as
U_{M−1}(cos(x))/M where U_k is the k'th
Chebyshev polynomial of
the second kind.

This is a very nice observation. There is one problem I am
having in
applying them to my problem. I need to know (or have a
decent bounds)
on the absolute value of the first extreme value of U_m(x)
after x=0.
So in the case of U_2(x), you could solve for the critical
points,
take the critical point closest to zero (which is not equal to
zero)
x*, and then compute |U2(x*)|. But of course for M>4, this
strategy
will not work, and there is no obvious way I can see to get a

(close)
upper bound to this first extreme value. (Could always do
this
computationally, but if there was a proven bound, that of
course would
be better).

|sin(Mx)/sin(x)| <= |1/sin(x)|

The peaks will be quite close to points where sin(Mx) = (+/−) 1,

Don't you mean just when M is odd?

No, both even and odd. The point is simply that sin(Mx) varies
rapidly while 1/sin(x) varies slowly (away from the points where sin(x)=0).
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