

Transitive Order of a Relation

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Given a relation R , which is a subset of the power set P of the Cartesian product $S \times S$ of a set S with itself, R is either transitive or it is not. The transitive closure of R is well defined and is conventionally noted R^t , where t is not an integer, but indicates "transitive".

Matrix multiplication may be used to find the transitive closure of a graph. The m th power of an adjacency matrix is the number of paths of length m between the relevant vertices. Each time the adjacency matrix A of a graph G of a relation R is squared, new paths of length two may appear. For large graphs this is easier using computer methods than looking for potential closures in the graph by inspection. When none appear, we are done; we have found the transitive closure of the relation.

By defining $0/0 = 0$, we may normalize any power of an adjacency matrix simply to directly represent the transitive closure of a graph. We may write (piecewise) $(R / |R|)$ to normalize a matrix if $0/0 = 0$. Then, the transitive order of a relation R is

the minimal power n such the normalized adjacency matrices

(piecewise)

$$((R^{(2^n)} / |R^{(2^n)}|) \cup (R^{(2^{n+1})} / |R^{(2^{n+1})}|)).$$

The transitive order of a transitive relation is 1.

The transitive closure of a relation with transitive order n is therefore

$$((R^{(2^n)} / |R^{(2^n)}|) \cup (R^{(2^{n+1})} / |R^{(2^{n+1})}|)).$$

Is this known?

Doug Goncz
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