Re: Transitive Order of a Relation

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* *From*: quasi <quasi@xxxxxxxx>
* *Date*: Wed, 09 May 2007 13:04:25 −0500

On 9 May 2007 02:52:43 −0700, DGoncz@xxxxxxxxxxxx wrote:

> On May 9, 4:58 am, quasi <q...@xxxxxxxx> wrote:
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> > On 8 May 2007 19:30:30 −0700, DGo...@xxxxxxxxxxxx wrote:
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> > > > On May 6, 8:33 am, DGo...@xxxxxxxxxxxx wrote:
> > > >
> > > > > Given a relation R, which is a subset of the power set P of the
> > > > > Cartesian product SxS of a set S with itself, R is either transitive
> > > > > or it is not. The transitive closure of R is well defined and is
> > > > > conventionally noted R^t, where t is not an integer, but indicates
> > > > > "transitive".
> > > > >
> > > > > Matrix multiplication may be used to find the transitive closure of a
> > > > > graph. The mth power of an ajacency matrix is the number of paths of
> > > > > length m between the relevant vertices. Each time the adjacency matrix
> > > > > A of a graph G of a relation R is squared, new paths of length two may
> > > > > appear. For large graphs this is easier using computer methods than
> > > > > looking for potential closures in the graph by inspection. When none
> > > > > appear, we are done; we have found the

transitive closure of the
relation.

By defining 0/0 = 0, we may normalize any power of an adjacency matrix simply to directly represent the transitive closure of a graph. We may write (piecewise) (R / |R|) to normalize a matrix if 0/0 = 0. Then, the transitive order of a relation R is

the minimial power n such the normalized adjacency matrices

(piecewise)
((R^(2^n)) / | R^(2^n)) |) = ((R^(2^(n+1))) / | R^(2^(n+1)))
|).

The transitive order of a transitive relation is 1.

The transitive closure of a relation with transitive order n is therefore

((R^(2^n)) / | R^(2^n)) |).

Is this known?

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> > Hello? Is anybody listening?
> >
> > Doug
>
> Please define the term "relation of transitive order n"
>
> At first glance, it appears you are defining the order of a relation to be the least positive integer n power such that your sequence of matrices stabilizes. If so your claim is true by definition −− there's nothing to prove.

Yes, thank you for replying, quasi.

OK, it's true by definition.

Is it of any use and might a journal article be of interest to others? Or perhaps a letter to the editor of a journal?

Is there a standard method to find the transitive closure of a complicated relation, say of order > 10, other than matrix powers.

I'll let other more knowledgeable people respond to this last question, but I think it's this question that you should have asked initially.

Also, I asked you a question which you never really answered, so I'll ask it again. How are you defining the transitive order of a relation? Can you give some simple examples to illustrate?

Is it a standard terminology? If so, are you using a standard definition?

I would have assumed the term transitive order to mean this ...

Start with a relation R, regarded R as a graph. For two elements a,b in the same connected component of R, define the distance from a to b, in the usual way, as the length of the shortest path from a to b. Then define the transitive order of R to be the maximum distance for all such pairs a,b. In other words, the transitive order of R is just the maximum diameter of the components of R.

With the definition I proposed above, a relation is transitive iff it has transitive order 1, which I think agrees with your interpretation. However, using this definition, starting with any positive integer n, it's easy to create a relation with transitive order exactly n. How would the method of successive squaring distinguish between a relation of transitive order 3 and one of order 4?

Some examples would be helpful.

quasi
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