Re: What's in a name?

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- *From*: Achimota <dgreenhoe@xxxxxxxxx>

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> > > "What is the normal mathematical name given to the set of
> > > domain points that lead to a range of zero?
> >
> > Null space?
>
> Or: kernel?

If the question is with regards to an operator (e.g. matrix, integral operator, or any other operator), then the answer is either "null space" or "kernel" (as has already been suggested by others). If the question is with regards to some other mathematical object, you can ignore the remainder of this post (of course you can ignore it either way ^__^)

In particular, suppose an operator A maps vectors from a vector space X to a vector space Y (often denoted A:X−−>Y).
Then the null space N of operator A is defined as
N(A) = { x in X | Ax = 0 }
("the set of all x in X such that Ax=0")

and the range space R of operator A is defined as
R(A) = { y in Y | y=Ax for some x in X }.
("the set of all y in Y such that y=Ax for some x in X")

These two sets, N(A) and R(A), are rightly called "spaces" because both are "linear subspaces" −−− that is, they are both vector spaces and their elements are vectors such that you can add any two vectors together, multiply any of them by a scalar to produce another vector in the set, etc.

The two sets are also important because of their part in the "fundamental theorem of linear equations":
dim N(A) + dim R(A) = dim X
where dim is the dimension of a subspace and X is a finite dimensional subspace and A:X−−>Y is a linear operator.

The "null space" is also called the "kernel" and the range is also called the "image" (Bollobas page 28). However, I (howbeit my opinion

not being so significant) would prefer not to use the term "kernel" in this sense because it is also the name used with the ever popular integral operators with "kernel" k(x,y) as in
[A f(x)](y) = int_0^1 k(x,y) f(x) dx
The Fourier transform is an example of an integral operator with kernel
k(t,w) = e^{−iwt} such that
[F f(t)](w) = int_t f(t) e^{−iwt} dt

References:

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I wish you all the best in your study,
Dan Greenhoe

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