Re: Sum of squares−

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- *From*: JEMebius <jemebius@xxxxxxxxx>
- *Date*: Sat, 19 May 2007 17:00:49 +0100

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chrizm7@xxxxxxxxx wrote:

> If a^2 + b^2 = x^2 + y^2 (and a, b, x, y all not zero)
>
> then I am 99% sure either a = x, b = y or a = y, b = x. But what makes this true, because it is clearly not true if we remove the squares:
>
> 2 + 5 = 1 + 6
>
> Geometrically, of course, a^2 + b^2 = r is the equation of a circle.
> So if x^2 + y^2 = r as well, both equations represent the same circle.
>
> But I would like a symbolic proof without appealing to geometry.

The subject of "Sums of Squares" is a mathematical universe in its own right.
Let me just recall just the two theorems that are most relevant to your observation.
Both are applicable to positive integers.

(A)
A prime number p = 4n+1 is the sum of two squares in an essentially unique way.

(B)
A number which is the product of N different prime numbers of the form 4n+1 is the sum of two squares in essentially 2^(N−1) different ways.

Therefore your conjecture is true (even 100% instead of 99%) only in the case that a^2 + b^2 is a prime number.

Literature:
Hardy and Wright: An introduction to the theory of numbers. Oxford, 1938; reprinted at least four times...

and many more books! But this one is among the best, perhaps the best introduction to number theory.

Happy studies: Johan E. Mebius
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