Re: x^2 − Ay^2 =1

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- *From*: "Philippe 92" <nospam@xxxxxxxxxxxx>
- *Date*: Sun, 20 May 2007 16:22:16 +0200

Vincenzo Librandi wrote :

> Philippe wrote:
>
> > We are waiting for your polynomials giving
> > A = 61, 109 or 421...
>
> 1766319049^2−61*226153980^2=1
> is tied

Hi,

I don't understand what you mean by 'tied'
tied to what ? related with the =−1 equation ?

Also, yes you have a solution to that specific equation, but
not with your method of polynomial representation for A !
(61 is not in the list of your allready given polynomial forms)

> 29718^2−61*3805^2=−1
> find the tie.

what tie ???
if X, Y is a solution of any generalized Pell's equation
x^2 − A*y^2 = −1, then
X^2 + A*Y^2, 2X*Y is a solution of
x^2 − A*y^2 = +1

This is true for all A for which x^2 − A*y^2 = −1 has a solution.

And generalized into :
If X,Y is solution of x^2 − A*y^2 = p
and U,V solution of x^2 − A*y^2 = q, then
U*X + A*V*Y, U*Y + V*X is solution of x^2 − A*y^2 = p*q

> For others: tied x^2−109*Y^2=−1
> or x^2−421y^1=−1, and I fiend what ask.

Again, I don't understand your 'tied'.
None of 109 and 421 is in your list of polynomials.
What do you mean by 'fiend' ?

And as above, if we know a solution of x^2 − A*y^2 = −1,
then we get immediately the solution for x^2 − A*y^2 = +1.
So *if* x^2−109*Y^2=−1 or x^2−421y^1=−1 have solutions (and they do)
we can deduce from these solutions the solution of the +1 equations.

But if the equation x^2 − A*y^2 = −1 has no solutions, and many
of them don't, we can deduce nothing about solution of the
corresponding x^2 − A*y^2 = +1

Also the problem is not in just solving these equations, but
to apply _your_ method in solving them.
That is find polynomial forms for A, X, Y which fit to this equation.

I have reordered your answers, the two equations are independent.

> > I give you a solution, which is may be or not the fundamental
> > one,
> > of this equation : x^2 − 21*y^2 = 1
> > X = 665335, Y = 145188
> > Could you say if this solution is fundamental or not ?
> > Could you find from this the fundamental solution ?
> > show me your 'simple decomposition'...

> Of course [...]
> 665335=(5,11,12097) isn't square
> so 55=(5,11)isn't square; and 55^2−21*12^2=1.

Usually just write 665335 = 5*11*12097 for prime decompositions,
writing (a,b,c...) is confusing.

> I find the metod !

What method ?
You fell into a lucky coincidence for the 55.

I would have given
X = 73180801, Y = 15969360 instead of the 665335, what would your "method" have given ?
73180801 = 17*31*138863, yes it is not a square, neither are
17*31 = 527
nor 17*138863 = 2360671
nor 31*138863 = 4304753
neither of these three being solutions of x^2 − 21y^2 = 1
although 73180801 _is_ a solution, and is not the fundamental solution. What do you deduce ???
The same if I had given X = 6049 = 23*263.

Yes 55 is the fundamental solution, that you didn't proove, although with y = 12 a proof could be just try and check all values
y = 1, 2... 11, or just solve the Pell's equation using classical methods, what you reject.

And all solutions of this equation (A=21) are, in increasing order :

x = 1, y = 0 (the trivial solution of all Pell's equations)
x = 55, y = 12 (the fundamental solution of this equation)
x = 6049, y = 1320
x = 665335, y = 145188
x = 73180801, y = 15969360
x = 8049222775, y = 1756484412
x = 885341324449, y = 193197315960
etc... (from allready discussed recurrence relations)
and there are _no_ others, just because we start from the fundamental one, apart changing x into −x and/or y into −y.

[reordered, this is the second equation, unrelated with the first]

> Another example
> x^2 − 85*y^2 = 1
> X = 285769, Y = 30996
> same question.

> Of course [... and] 285769 = (11,83,313) isn't square

Do you mean that 913 = 11*83 should be a solution of
x^2 − 85*y^2 = 1 ? or just 11, or 83 ... ?
Did you proove that 285769 is or not the fundamental solution for this equation ?

A _method_ is supposed to work in all cases, or should allways
be able to distinguish between cases where it applies and cases
where it doesn't !

Regards.

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