Re: x^2 − Ay^2 =1

Source: http://sci.tech−archive.net/Archive/sci.math/2007−05/msg04171.html

- *From*: "sttscitrans@xxxxxxxxx" <sttscitrans@xxxxxxxxx>
- *Date*: 24 May 2007 08:02:50 −0700

On 24 May, 00:51, "Philippe 92" <nos...@xxxxxxxxxxxx> wrote:

> sttscitr...@xxxxxxxxx wrote :
>
>> On 22 May, 10:08, "Philippe 92" <nos...@xxxxxxxxxxxx> wrote:
>
> If you say that... I don't have this book.
> and yes, _binary form_
> not "any real number".

Yes, but the point is that you can solve say

x^3 − 19y^3 =−1
without leaving the integers, by transforming
the equation.

You can arrive at the same result by expanding
r = cubrt(19) as a CF.

Of course you do not get an infinite number
of solutions from 8+3r as the powers of 8+3r
are ternary, not binary units. So a ternary
form F(x,y,z) with F(x,y,z)F(x',y'z') = F(x",y"z")
would have an infinite number of solutions.

>> It is also one of the methods that Dario Alpern
>> used to implement his solver for
>> AX^2 +BXY +CY^2 = N
>
> yes but ...
> abstract from Dario Alpern's program :
> That is Dario Alpern doesn't use the "general" algorithm but the

> integer variant, whatever the name you give to it.
> Variant which is suitable only for quadratic numbers.
> (that is any number in Q(sqrt(d)), for any d>0 not a square)

It should be on his methods page.

> (the original problem was to solve the normal Pell's equation
> x^2 − A*y^2 = 1 in Z, using A, x, y = polynomials in n.
> For any n, p(n) being an integer, p is in Z[X], not Q[X].
> Searching for solutions in Q[X] is equivallent to searching for
> _rational_ x, y in a Pell's equation...)

sqrt(n^2 +n +1) = an +b + c/n + d/n^2 + ..

The '"integer part" of the expression can be thought of
as an +b the fractional part" c/n + d/n^2 + ..

You can use synthetic division to obtain
1/(c/n + d/n^2 +) = a'n +b' + c'/n + ...

The coefficients are rational in general.

The convergents in this CF approximate
in some sense sqrt(n^2+n+1)

[P(n)/Q(n)]^2 a.e. n^2+n+1

It is another matter whether the approximation is good enough to give
x^2 −(n^2+n+1)y^2 = 1 or −1

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