Re: closed sets, limit points

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- *From*: frank <frank.degeeter@xxxxxxxxx>
- *Date*: 28 May 2007 09:32:01 −0700

On 28 mei, 19:18, quasi <q...@xxxxxxxx> wrote:

> On 28 May 2007 07:02:43 −0700, frank <frank.degee...@xxxxxxxxx> wrote:
>
>> A limit point is a point every deleted neighbourhood of which contains at least one point of C. Now, it is possible that each of these neighbourhoods contain elements outside C as well. In that case, the limit point is a boundary point. If not, some neigbourhoods are completely contained in C. The limit point then is an internal point.
>
> For future reference, please don't top post.
>
>> José Carlos Santos schreef:
>>
>>> On 28−05−2007 11:01, frank wrote:
>>>
>>>> A set C in R^n is closed if and only if it
>>>> contains all its limit
>>>> points. My analysis book has a proof of this
>>>> theorem. While pondering
>>>> over it, I possible devised another proof, but
>>>> I would be thankful if
>>>> someone would have a look at it and tell me
>>>> whether it is correct − I
>>>> am suspicious because it looks a bit too
>>>> simple.
>>>>
>>>> Here it goes:
>>>> If the set is closed, by definition it contains
>>>> all its boundary

points. But any boundary point is also a limit
point. Limit points
which are not boundary points are internal
points, and therefore also
contained in the set. Therefore, it the set is
closed, it contains all
its limit points.
If a set contains all its limit points, it also
contains its boundary
points, because these are limit points as well.
But then, by
definition, the set is closed.

Right?

How do you prove that "[l]imit points which are not
boundary points are
internal points"? Not that I doubt that it is true.

Best regards,

Jose Carlos Santos

Your proof looks fine to me.

Note, a closed set is often defined as a set which contains all its
limit points. One can then prove (in the same way as you did) that
such a definition is equivalent to the one in your book.

quasi

Thanks very much!
Frank

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