# Re: Proof 0.999... is not equal to one.

***Source:*** http://sci.tech−archive.net/Archive/sci.math/2007−05/msg05392.html

- *From*: Rupert <rupertmccallum@xxxxxxxxx>
- *Date*: 31 May 2007 00:02:46 −0700

On May 31, 4:16 pm, chaja...@xxxxxxxx wrote:

> I have written a proof that 0.999... cannot be equal to one in the
> system of real numbers.
>
> While at the end of it all you may not fully agree with my proof, much
> I as have never seen a proof asserting they were equal that I was able
> to consider valid, I'm sure you will agree that the
> ideas I present are not a simply rehashing of basic objections of
> others before me.
>
> It is available in several
> formats:http://www17.brinkster.com/chajadan/Math/Proofs/Proof1.dochttp://www17.brinkster.com/chajadan/M
>
> −−Charles J. Daniels
> chaja...@xxxxxxxx

I can't gain access to that webpage.

Here are the generally accepted axioms for the real numbers:

(1) For any real numbers a, b, and c, a+(b+c)=(a+b)+c
(2) For any real numbers a, b, a+b=b+a
(3) There exists a unique number 0 such that for all numbers a, a+0=a
(4) For all numbers a there exists a unique number −a such that a+(−
a)=0
(5) For all real numbers a, b, c, a.(b.c)=(a.b).c
(6) For all real numbers a, b, a.b=b.a
(7) There exists a unique number 1, different from 0, such that for
all
numbers a, a.1=a
(8) For all numbers a different from 0 there exist a unique number
a^(−1) such that a.a^(−1)=1
(9) For all numbers a, b, c, a.(b+c)=(a.b)+(a.c)
(10) For all numbers a, b, if a>0 and b>0, then a.b>0
(11) For all numbers a, b, if a>0 and b>0, then a+b>0
(12) For all numbers a, b, a<b if and only b>a
(13) For all numbers a, b, c, if a>b, then a+c>b+c

(14) For every set S of real numbers, if S is nonempty and there exists
a number a such that for all x in S, a>x or a=x, then there exists a number b with that property such that, for every number a with that property, b<a or b=a. The number b is called the least upper bound of S.

We define 10 to be 1+(1+(1+(1+(1+(1+(1+(1+(1+1))))))))).

We define S to be the set of all real numbers x which are in every set T
with the property that 10^(−1) is in T, and whenever y is in T, y. 10^(−1)
is also in T. Then we define S' to be the set of all numbers x such that
1−x is in S. Thus S={0.9, 0.99, 0.999, ...}.

We can prove from the above axioms that there exists a unique number x such that x is an upper bound for S (i.e. is greater than or equal to every
member of S) and is less than or equal to 1. We call this number 0.9999....

It can be proved from the axioms that this number is also equal to 1.

Thus 0.9999...=1. There is certainly no doubt that this follows from the axioms given using second−order logic. I can show you the details if you like.

If your proof also uses these axioms, then you've shown that the axioms
are inconsistent, but I don't think this is very likely. It's a shame I
can't see your proof and show you the mistake.

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