

# Re: Dedekind Cuts, Fundamental Sequences: why?

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On Sun, 03 Jun 2007 22:57:37 -0400, Hatto von Aquitanien <abbot@xxxxxxxxxxxxxxxx> wrote:

Bob Kolker wrote:

Hatto von Aquitanien wrote:

What is the step of logic which leads one to seek an extension of the rational numbers to the real numbers?

Very simple. You want every set of numbers bounded from below to have a greatest lower bound and every set of numbers bounded from above to have a least upper bound. While rational numbers are dense in their ordering they lack the closure of boundedness, hence real numbers are invented to extend the rationals.

This is the motivation stated by Pickert and Görke, but it is not clear to me why it matters.

The fact that the reals are complete is used in many places – analysis simply would not work without it. You're just at the start...

Nor is it completely clear what it means to say that the field of rational numbers does not exhibit the closure property of boundedness.

Say  $S$  is the set of positive rational  $x$  such that  $x^2 < 2$ . Then  $S$  does not have a rational least upper bound.

Re: Dedekind Cuts, Fundamental Sequences: why?

For every rational number I can very easily divide the rational numbers into two disjoint sets by asserting that every rational number greater than the selected number is a member of the set whose lower bound is the selected number. Likewise for the symmetrically opposite case. I then arbitrarily chose one side of the bifurcation to include the chosen rational number.

Every definition I have consulted for supremum and infimum begins with the real numbers. So to tell me that the reason we need to extend the rational numbers to the real numbers is so that the domain of numbers has suprema and infima assumes the real numbers to be defined already.

Huh?

Definition: The ordered field  $F$  is complete if every nonempty subset of  $F$  which is bounded above has a least upper bound.

That definition does not begin with the real numbers. If you've only seen the definition in the context of the real numbers that's because the reals are complete, while, say, the rationals are not. It does not follow that "to say that the reason we need to extend the rational numbers to the real numbers is so that the domain of numbers has suprema and infima assumes the real numbers to be defined already."

[http://www.cuyamaca.edu/bruce.thompson/Fallacies/circ\\_justification.asp](http://www.cuyamaca.edu/bruce.thompson/Fallacies/circ_justification.asp)

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