Re: Dedekind Cuts, Fundamental Sequences: why?

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- *From*: David C. Ullrich <ullrich@xxxxxxxxxxxxxxxxx>
- *Date*: Mon, 04 Jun 2007 04:54:38 −0500

On Mon, 04 Jun 2007 05:37:27 −0400, Hatto von Aquitanien
<abbot@xxxxxxxxxxxxxx> wrote:

> David C. Ullrich wrote:
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>> On Sun, 03 Jun 2007 22:57:37 −0400, Hatto von Aquitanien
>> <abbot@xxxxxxxxxxxxxx> wrote:
>>
>>> Bob Kolker wrote:
>>>
>>>> Hatto von Aquitanien wrote:
>>>
>>> This is the motivation stated by Pickert and Görke, but it is
>>> not clear to
>>> me why it matters.
>>
>> The fact that the reals are complete is used in many places − analysis
>> simply would not work without it. You're just at the start...
>
> That begs the question.

Yes it does. When you know some analysis it will be very clear
to you why completeness is so important.

>>> Nor is it completely clear what it means to say that
>>> the field of rational numbers does not exhibit the closure
>>> property of
>>> boundedness.

> > Say S is the set of positive rational x such that x^2 < 2. Then
> > S does not have a rational least upper bound.

> How does that add anything to the already established fact that x^2=2 does
> not have a rational solution?

Make up your mind! A paragraph up you complain that I didn't answer
your question. Here I _did_ answer your question and you ask how
my answer adds to something else.

You _said_ it was not clear to you why the rationals were not
complete. I explained exactly why the rationals are not complete −
I didn't say anything about what I was or was not adding anything to.

> > > For every rational number I can very easily divide the
> > > rational numbers
> > > into two disjoint sets by asserting that every rational number
> > > greater
> > > than the selected number is a member of the set whose lower
> > > bound is the
> > > selected
> > > number. Likewise for the symmetrically opposite case. I then
> > > arbitrarily
> > > chose one side of the bifurcation to include the chosen
> > > rational number.
> > >
> > > Every definition I have consulted for supremum and infimum
> > > begins with the
> > > real numbers. So to tell me that the reason we need to extent
> > > the
> > > rational numbers to the real numbers is so that the domain of
> > > numbers has
> > > suprema and infima assumes the real numbers to be defined
> > > already.

> > Huh?
> >
> > Definition: The ordered field F is complete if every nonempty
> > subset of F which is bounded above has a least upper bound.

> That is not a definition of least upper bound.

I didn't say it was! It's a definition of completeness. You
know the definition of least upper bound, or I thought

you did.

> That definition does not begin with the real numbers. If you've
> only seen the definition in the context of the real numbers
> that's because the reals _are_ complete, while, say, the
> rationals are not. It does not follow that "to say that the reason
> we need to extend the rational numbers to the real numbers is so
> that the domain of numbers has suprema and infima assumes the
> real numbers to be defined already."
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> http://www.cuyamaca.edu/bruce.thompson/Fallacies/circ_justification.asp
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> ************************
>
> David C. Ullrich

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David C. Ullrich
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