Re: Dedekind Cuts, Fundamental Sequences: why?

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* *From*: Hatto von Aquitanien <abbot@xxxxxxxxxxxxxx>
* *Date*: Mon, 04 Jun 2007 21:25:47 −0400

James Burns wrote:

> Hatto von Aquitanien wrote:
>
> > David C. Ullrich wrote:
> >
> > > On Mon, 04 Jun 2007 05:37:27 −0400, Hatto von Aquitanien
> > > <abbot@xxxxxxxxxxxxxx> wrote:
> > >
> > > > David C. Ullrich wrote:
> > > >
> > > > > The fact that the reals are
> > > > > complete is used in many
> > > > > places − analysis simply
> > > > > would not work without it.
> > > > > You're
> > > > > just at the start...
> > > >
> > > > That begs the question.
> > >
> > > Yes it does. When you know some analysis it will be very
> > > clear
> > > to you why completeness is so important.
> >
> > That still begs the question. The fact that something _is_
> > useful does not explain _why_ it is useful.
>
> Please excuse me for butting in here, Dr. Ullrich.

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> Whether it begs the question depends on which question is on the table. Judging from the subject header, your first question seems to have been "Why are Dedekind cuts used to define reals?" (And partitions of Cauchy sequences as well?) That question seems to have been answered well, by several people, not just Dr Ullrich.

<quote>
What is the step of logic which leads one to seek an extention of the rational numbers to the real numbers?

I understand the arguments given by Prickert and Görke for all of the previous extentions. But when they go from the rationals to the reals, they don't really present a formal equation in need of a solution as they had for all of the prior extentions. It leaves me to wonder what, exactly, the criteria for success is.
</quote>

I still don't believe I have a good answer to my question. People presented examples such as the solution to x^2=2 as an equation in need of solution. As I have already pointed out, that is a single example, and not a general form. This is different from all of the previous steps in the development where there was a clearly stated formula which was in need of solutions not provided by the domain of numbers already developed. I'm presented with three ways of defining the real numbers. One is to use infinite decimal representation. The second is Dedekind cuts, and the third is fundamental sequences.

I am assured by the authors that proofs using the first definition are prohibitively lengthy. That leaves them to expound on Dedekind cuts and fundamental sequences. I'm told that ensuring every set of rational numbers has a supremum will "plug all the holes" in the rational numbers. I don't see how that is different from saying "if there's a hole, fill it with a real number". Why does {x:x \in \Q and x<sqrt(2)} not have a supremum in \Q? "There's a hole at sqrt(2)."

As I have already stated, the motivation for either of these two approaches appears to be so that the algebraic features of the field of rational numbers can be carried over to the real numbers. Weyl flat out rejected the approach using terms such as "nonsense", "completely wrong", "not even a shadow of a proof" and "blatant circulus vitiosus".

> Because it is too, too convenient to give up saying these two algorithms calculate the same number and imagining that number as a dimensionless, structureless point on the real number line, we do say that they

> calculate "the same number". What is this number they
> calculate? In my view, that dimensionless, structureless
> point is "really" the set of all the good algorithms
> that calculate its position, which is to say, the equivalence
> class of Cauchy sequences that converge to that same point.
>
> And what is that point again? The Cauchy sequences
> that converge to it. It all seems very circular, but
> built up, step by step, from rationals, to sequences
> of rationals, to Cauchy sequences, to partitions of
> Cauchy sequences, it's all well−founded and consistent.

Not too long ago I proposed a possible means of defining the real numbers as a set of algorithms which are left in symbolic form until there is a need to arrive at numerical results. I didn't pursue it very vigorously because it seems like more work than it's worth.

I don't particularly like the way Weyl formulated vector spaces in what has now become received dogma. I can only interpret his reasoning to be that he rejected the extention of rational number multiplication to the real numbers, in general. There is nothing in the original concept of a finite dimensional vector space formed of n−tuples that would preclude an extention from rational to real component addition thence to multiplication if such were deemed possible for "scalars".

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