Re: Dedekind Cuts, Fundamental Sequences: why?

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- *From*: Hatto von Aquitanien <abbot@xxxxxxxxxxxxxx>
- *Date*: Wed, 06 Jun 2007 00:08:20 −0400

Dave Seaman wrote:

> On Tue, 05 Jun 2007 09:57:58 −0400, Hatto von Aquitanien wrote:
>
> > Aatu Koskensilta wrote:
> >
> > > On 2007−06−05, in sci.math, Hatto von Aquitanien wrote:
> > >
> > > > That tells me how they are useful, not why.
> > > > But what I really meant by
> > > > my previous comment is that he was begging
> > > > the question as to why
> > > > fundamental
> > > > sequences or Dedekind cuts are useful in
> > > > defining completeness. And I
> > > > still do not know if there was a clearly
> > > > defined objective which can
> > > > subsequently be verified as being
> > > > accomplished.
> > >
> > > They provide a concrete way of specifying a complete
> > > ordered field with
> > > a countable dense subset, reducing talk about reals to talk
> > > about
> > > naturals and sets of naturals.
> >
> > That is where Weyl rejected the development. He basically argues that
> > using sets of rational numbers as a means of defining the real numbers is
> > not
> > supported by any form of proof that it is even meaningful. It is also
> > fairly clear that he doubts such a proof could be given.

> I am certainly no expert on Weyl, but your description implies that he considered a "meaningful proof" to be something more than merely a *valid* proof. What is the difference, exactly?

I guess this is evidence that I should refrain from posting after studying for more than 20 hours without a break. I meant to write "supported by any form of proof, _or_ that it is even meaningful"

> > It is not completely clear to me that these definitions do reduce all discussion of real numbers to discussions of rational numbers. In places where I can perceive discussions of real numbers being derived strictly from concepts related to rational numbers, I don't see either fundamental sequences nor Dedekind cuts add anything essential, or even useful.
>
> I don't follow this. Which of the following are you claiming?
>
> (1) The constructions mentioned do not succeed in producing a complete ordered field, or
> (2) The notion of completeness is not essential or useful in mathematics?

Neither one. I was speaking "places where I can perceive discussions of real numbers being derived strictly from concepts related to rational numbers".

> > Such a reduction is of importance in that, provided
> > we pretty much agree what sort of principles and modes of reasoning are
> > acceptable when dealing with naturals and sets thereof, we get a clearer
> > view of what sort of reasoning about reals, functions of reals, and so
> > on, is acceptable.
>
> > That is another aspect of Weyl's argument. He claims the reasoning used to extend to rational numbers to the real numbers is cyclical.
>
> Cyclical? Do you mean circular?

Yes. I guess I translated the Latin incorrectly due to fatigue.

> How so?

"It is probably not necessary to repeat that it would be meaningless to include among these principles an assertion such as the following: If A is a property of properties, then one may form that property P_A which belongs to an object x if and only if there is a property constructed by means of these principles and itself possesses the property A. That would be blatant circulus vitiosis; yet our current version of analysis commits this error and I consider this ground for censure."

> Are you aware that the rationals are defined in terms of the integers, and the integers in terms of the natural numbers?

I have addressed that multiple times in this thread.

> Do you feel that those constructions are circular, as well?

I was citing Weyl. But see below.

> Do they add anything essential or useful to mathematics?

I have attempted to indicate that these extentions appear to be of a signally different nature from the extention from the rational numbers to the real numbers.

> > For myself,
> > if I do allow for some arguments founded in terms of the rational numbers to indicate that the rational numbers are in need of extention, it is not clear to me that all such circumstances are, or can be addressed in this way. For example, I can give an algebraic argument for wanting a solution
> > to 2=x^2. The motivation, at least, arrises from within the context of the
> > rational numbers. Pi, OTOH, seems to be a bird of a different feather.
>
> The motivation for completing the rationals lies mainly in analysis, not algebra.

My point was that using the algebra of the existing domain I can construct

expressional forms which sometimes, but not always produce a number within the existing domain. That was the line of development which led to each previous extension.

> Consider the mean value theorem, for example.

That hasn't been introduced yet. It's covered in volume II.

> Its proof
> depends heavily on the completeness of the reals, and this theorem lies behind much of analysis.

Why not just extent the natural numbers immediately to the positive real numbers using infinite decimal expression, introduce 0 and negation and be done with it?

I am assured that development of the algebra of real numbers is excessively unwieldy when approached in this way, but I have not seen that demonstrated.

> > There are constructs which we perceive to warrant inclusion in the set of things we call numbers. IOW, when we view the field of rational numbers, and compare it with our expectations of completeness, it comes up short. We can identify constructs which lie outside the set which we want to treat
> > as numbers. Fundamental sequences and Dedekind cuts appear to me to be simply an elaborate loophole set up to allow the inclusion of everything our intuition tells us aught to be in the set of numbers (excluding Gaussian numbers, which is another can of worms, or the same can from the other end.)
>
> Is the construction of the rationals from the integers also an enormous loophole, in your mind? Please explain.

No. Each of those steps was taken after explictly and clearly stating the objective.

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