Re: representation

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- *From*: quasi <quasi@xxxxxxxx>
- *Date*: Wed, 06 Jun 2007 22:20:23 −0500

On Wed, 06 Jun 2007 17:29:09 −0700, slapjack777@xxxxxxxxxxx wrote:

> On Jun 6, 3:37 pm, tommy1729 <tommy1...@xxxxxxxxx> wrote:
>
> > > > every positive integer is the sum of at most 8
> > > > squared primes (including 1)
> > > >
> > > > tommy1729
> > >
> > > cmon people show your math skills :−)
> > >
> > > not even a crankpot gonna try ??
> >
> > nobody this year ?
>
> http://mathworld.wolfram.com/LagrangesFour−SquareTheorem.html

To say that every positive integer is a sum of 4 squares is not the same as saying every positive integer is the sum of at most 8 squares of primes (including 1).

Tommy1729's sum of 8 squares of primes conjecture is a reasonable claim, but needs some numerical testing.

Of course, a single counterexample would kill it instantly.

Another way to disprove the conjecture without producing an explicit counterexample would be to show that the density of the set of numbers which can be represented as sum of at most 8 squares of primes (including 1) is less than 1.

On the other hand, if the conjecture is true, proving it might be near impossible.

However, even if 8 doesn't work, it's intuitive that it will work for _some_ positive integer k in place of 8.

So here's my more modest version of Tommy1729's conjecture ...

Conjecture (1):

For every positive integer n, there is a positive integer k, depending only on n, such that every positive integer is a sum of at most k n'th powers of primes (where we include 1 as a prime).

Here's an even weaker conjecture ...

Conjecture (2):

For every positive integer n, there is a positive integer k, depending only on n, such that the density of the set of positive integers which can be represented as a sum of at most k n'th powers of primes (where we include 1 as a prime) is 1.

Remarks:

Clearly, if conjecture (2) fails for a given n, then it fails for all higher values of n. Moreover, if conjecture (2) fails for a given n , then conjecture (1) also fails for that n.

For n=1, both conjectures are almost certainly true, and are perhaps instantly implied by known results.

Moreover, for n=1, since k is free to be any fixed positive integer, there might be really elementary proofs, at least for conjecture (2).

Of course, for n=1, the truth of Goldbach's conjecture would imply that k=3 suffices (and hence is best possible) for conjecture (1).

Note that for n=2, Tommy1729 claims that k=8 suffices for conjecture (1), whereas I only claim that some k suffices.

quasi
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