Re: Proof of Dirichlet's Test for convergence of given integral

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- *From*: precarion <precarion@xxxxxxxx>
- *Date*: Thu, 07 Jun 2007 08:24:05 EDT

Wow... Your proof is fantastic! I was thinking about Riemann Integrals, but you have provided the proof that can be used for Lebesgue Integrals as well. There is only one problem... I need a proof that can be easily explained to the studens of the 1st year of mathematics... On my university the measure theory is on the 2nd year of studies, so I believe that your proof unfortunately (I really like it!) will be too hard for them to understand.

Do you possibly know any proof of Dirichlet's Test for convergence of integrals that is using only Riemann Integrals? (I've already found one in Fichtenholtz's calculus book, but it's too boring in my opinion, and I'm still looking for something else...)

Thanks,
Chris

> On Wed, 06 Jun 2007 07:30:18 EDT, precarion
> <precarion@xxxxxxxx>
> wrote:
>
> > Hello Everyone!
> >
> > I was looking for a proof of the Dirichlet's Test
>
> for convergence of integrals throught all of my
> books, but without much luck... Can somebody help me
> by writing down the proof on the forum or by pointing
> out some Internet resources that include the proof?
>
> > THEOREM *Dirichlet's Test*
> > "If f(x) and g(x) satisfy the following conditions:
> > (a) integral from a to u of f(x)dx for a <= u < +oo
>
> is bounded,
>
> Of course we need some hypothesis to guarantee that
> int_a^u f(x) dx _exists_. Below I'm going to assume
> that f is continuous on [a, infinity).

(b) g(x) is monotonic on [a, +oo) and lim g(x) = 0,

when x −> +oo,

then the integral from a to +oo of (f(x)*g(x))dx is

convergent."

This follows by integration by parts. That would be
integration
by parts for Stieltjes integrals. I wouldn't get the
details
straight if I stated it in those terms − here's a
more or less
equivalent proof stated in terms of measures:

First, by changing the value of g at countably many
points
we can assume that g is right−continuous. The
modified
g is still monotone, we haven't changed the value of
any of the integrals.

Now it follows that there exists a finite positive
measure mu
such that

g(x) = mu((x,infinity)).

Let chi_x be the function with chi_x(t) = 1 for t >
x,
chi_x(t) = 0 for t <= x. And let F(u) = int_a^u f(x)
dx,
so we're assuming that F is bounded.

So for b > a we have

int_a^b f(x) g(x) dx = int_a^b f(x)
x) mu((x,infinity)) dx

= int_a^b f(x) int_a^infinity chi_x(t) d mu(t)
d mu(t) dx

= int_a^infinity int_a^b chi_x(t) f(x) dx d
x) dx d mu(t)

= int_a^infinity int_a^{min(t,b)} f(x) dx d
x) dx d mu(t)

= int_a^infinity (F(min(t,b) − F(a)) d mu(t).

Now, for each t we have lim_{b−>infinity} F(min(t,b))
= F(t).
Since F is bounded and mu is a finite measure, the
dominated
convergence theorem shows that the integral above
tends to

int_a^infinity (F(t) − F(a)) d mu(t)

as b −> infinity, QED.

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David C. Ullrich

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