

# Re: Proof of Dirichlet's Test for convergence of given integral

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*Source:* <http://sci.tech--archive.net/Archive/sci.math/2007-06/msg01516.html>

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  - *Date:* Fri, 08 Jun 2007 04:49:29 -0500
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On Fri, 08 Jun 2007 00:52:28 +0100, Timothy Murphy  
<tim@xxxxxxxxxxxxxxxxxxxxxxxx> wrote:

precarion wrote:

Wow... Your proof is fantastic! I was thinking about Riemann Integrals, but you have provided the proof that can be used for Lebesgue Integrals as well. There is only one problem... I need a proof that can be easily explained to the students of the 1st year of mathematics... On my university the measure theory is on the 2nd year of studies, so I believe that your proof unfortunately (I really like it!) will be too hard for them to understand.

Do you possibly know any proof of Dirichlet's Test for convergence of integrals that is using only Riemann Integrals? (I've already found one in Fichtenholtz's calculus book, but it's too boring in my opinion, and I'm still looking for something else...)

I would have thought the simplest way would be to first prove the corresponding result for series  $\sum a_n b_n$ , which is well-known and useful in many cases (eg in studying the convergence of Dirichlet series  $\sum a_n n^{-s}$ ).

If in fact the integrals involved were all Riemann integrals then the series result would extend at once to the integral result, using the usual approximation to the integrals by sums.

Is this entirely clear? It may well be so, and in fact it may well be trivial, but it's not entirely clear to me that it's going to work with no problem (not that I've tried writing it down carefully).

What bothers me is that an \_improper\_ Riemann integral

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is not a priori approximated by a Riemann sum, it's by definition the limit of integrals over compact intervals, and \_those\_ integrals are approximated by Riemann sums. So it seems like there's going to be an interchange of limits at some point, and if swapping two limiting operations always worked with no problem most theorems of analysis would be trivial.

Seems possible to me that it works, but that it may not work just by applying the result for sums per se, rather one is going to need to insert the proof of the result for sums and keep track of the epsilons. (???)

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David C. Ullrich

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