Re: Proof of Dirichlet's Test for convergence of given integral

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- *Date*: Sat, 09 Jun 2007 06:29:16 −0500

On Fri, 08 Jun 2007 13:43:08 +0100, Timothy Murphy
<tim@xxxxxxxxxxxxxxxxxxxxxx> wrote:

> David C. Ullrich wrote:
>
> > > Do you possibly know any proof of
> > > Dirichlet's Test for convergence of
> > > integrals that is using only Riemann
> > > Integrals? (I've already found one
> > > in Fichtenholtz's calculus book, but it's too
> > > boring in my opinion, and
> > > I'm still looking for something else...)
> >
> > I would have thought the simplest way would be to first
> > prove
> > the corresponding result for series sum a_n b_n ,
> > which is well−known and useful in many cases
> > (eg in studying the convergence of Dirichlet series sum a_n
> > n^{−s}).
> >
> > If in fact the integrals involved were all Riemann integrals
> > then the series result would extend at once to the integral
> > result,
> > using the usual approximation to the integrals by sums.
>
> Is this entirely clear? It may well be so, and in fact it may
> well be trivial, but it's not entirely clear to me that it's
> going to work with no problem (not that I've tried writing
> it down carefully).
>
> What bothers me is that an _improper_ Riemann integral
> is not a priori approximated by a Riemann sum, it's by
> definition the limit of integrals over compact intervals,
> and _those_ integrals are approximated by Riemann sums.

> So it seems like there's going to be an interchange of limits at some point, and if swapping two limiting operations always worked with no problem most theorems of analysis would be trivial.
>
> Seems possible to me that it works, but that it may not work just by applying the result for sums per se, rather one is going to need to insert the proof of the result for sums and keep track of the epsilons. (???)

I would have thought it was easy enough ...

To show I(a,infty) converges, one has to show that
I(a,b)−>0 as a,b−>infty, ie given eps there is a C such that

|I(a,b)| < eps if a,b >