Re: * says: Definition: sum{i in N} i = 0

Source: http://sci.tech−archive.net/Archive/sci.math/2007−06/msg02000.html

- *From*: Franziska Neugebauer <Franziska−Neugebauer@xxxxxxxxxxxxxxxxxxx>
- *Date*: Mon, 11 Jun 2007 01:07:37 +0200

WM wrote:

> On 10 Jun., 15:52, Franziska Neugebauer <Franziska−
> Neugeba...@xxxxxxxxxxxxxxxxxxx> wrote:
>
> > WM backlog:
> >
> > A
> >
> > > "Divergent series" means: There is no limit, sum, value in
> > > the real
> > > numbers.
> >
> > "No limit" precisely means there is no L in M having the properties
> > given inhttp://en.wikipedia.org/wiki/Divergent_sequence.
> > "Divergent series" means: the series has no limit.
> >
> > Accept or deny?
>
> Of course accepted, although you probably should not gather all your
> mathematical knowledge from Wikipedia alone, in particular not from
> the German Wikipedia.

I either do not gather it from third−rate literature recently referred
to by you.

> If you add the fact that the limit of a series
> is the sum of its terms,

There is a subtle difference in the correct wording: The value of an
infinite series is _defined_ to be the limit of the partial sums of its
underlying infinite sequence _if_ the limit exists. Otherwise there is

no value. In that case the (value of the) series is undefined.

> then you get the result that a divergent
> series has no sum in R. It is easy to prove that it cannot have a sum
> in R.

You are again in error showing that you do not really accept the definition in wikipedia: The correct wording is: A divergent series has no value (= is undefined) if there is no limit of the partial sums of its underlying sequence. Independently from the limit definition the divergent series has no value on its own right unless one defines its value.

> Therefore it is easy to prove that Dik's definition is wrong.

> From the correct definition it follows that everybody is free to define

the value of the divergent series ad lib.

B

Dik's definition (assignment of the value 0 to
a non−convergent
series)does neither contradict to

1) every natural number is positive, nor
2) that a finite sum of naturals is larger than
any of its
summands, nor
2a) that an infinite sum of naturals (> 0) has
no limit in N, nor
3) that the infinite sequence of naturals is
divergent in N.

This shows: You have no clue of logic conclusions.

If you want to claim that Dik's definition does contradict to 1), 2), 2a) or 3) you have show that.

> If you believe to be the chambermaid of Hilbert's hotel, then I cannot convince you, by any proof, that this hotel does not exist. Therefore I abstain from doing so.

Let me assure you that I would never ever check in into Hilbert's hotel not even in room #1. LOL Nonetheless you have to prove your claim.

> WM wrote:
>
> > WM wrote:
> >
> > > On 9 Jun., 10:44, Franziska Neugebauer <Franziska−Neugeba...@xxxxxxxxxxxxxxxxxxx> wrote:
> > >
> > > > WM wrote:
> > > >
> > > > > A sequence (a_n | n in N) of positive terms a_n has the (improper) limit oo, if the sequence (1/a_n) exists and has the limit 0.

This is
certainly a
possible
assignment
but it is no
longer in
the original
range of the
series (same
applies to
H&J).

It is completely in the
original range.

No. "oo" is certainly _not_ in the original
range c= N.
There is no element named "oo" in N or R.

But 0 is certainly in this range, and the method of reciprocals
is
allowed.

Read what you wrote! You had written

| A sequence (a_n | n in N) of positive terms a_n has the
| (improper) limit oo, if the sequence (1/a_n) exists and has the
| limit 0.

I said that

"oo" is certainly _not_ in the original range c= N
because there is no element named "oo" in N or R.

Now you wrote

| But 0 is certainly in this range, and the method of reciprocals
| is allowed.

1. This is not an argument at all since it is not 0 but you claim
"oo" to be the "(improper) limit" of a divergent series.

And I claim that this limit validly can be calculated by the

> reciprocals. The reciprocals have the limit 0 which is in R.

Irrelevant. BTW: 9 lines below I'll present the calculation of the reciprocals.

>> 2. In R there is no reciprocal of 0. It is meaningless to say "oo" were the reciprocal of 0 in the context of R and you know that only too well.

> Concerning limits it is correct to say that a sequence has the (iproper) limit oo if the sequence of reciprocals has the limit 0.

Dik's definition involves the divergent _series_

sum_{i e N} i,

i.e.

lim sum_{i=0}^n i
n -> oo

The sequence of reciprocals of the partial sums of his series is

(1 / sum_{i=0)^n i)_{n e N}

Already the first value (n = 0) of this sequence of reciprocals is undefined. OK nitpicking. Let's drop it for simplicity. According to you we shall now define, if I get you right:

1
lim sum_{i=0}^n i := ------------------------ = 1 / 0
n -> oo lim (1 / sum_{i=0)^n i)
n->oo

Q: How do we call the value of "1/0"?
A: "1/0" is undefined.
Q: Is "1/0" an element of R or N?
A: No.
Q: Does the method of the reciprocals shed any light on some "intrinsic value of Dik's sum" which is hidden by the series' divergence?
A: No.
Q: Does it prove Dik's definition wrong?
A: No.
Q: So what?
A: Good question.

> The only condition required here is that no term of the sequence is 0,

The first term *has* value 0, unfortunately.

> because then the reciprocal does not exist in R. But this condition is satisfied for the sequence of partial sums of N.

If we want to be ruthless we must concede that you are wrong.

> At least, I teach this so. Am I in error?

Im sorry to tell you that you are in this particular case.

A series can be treated as the sequence of its partial sums.

The value of an infinite series _is_ _defined_ as the limit value of its partial sum. If there is no L in M there is no such limit. In this case we say: The infinite series does not

converge.
Period.

The value or sum or limit of a series is the limit of its sequence of partial sums and that is an extended definition of the sum of its terms.

? (possibly too many or's)

Rather too less,

Keep it Simple, Sweatheart (KISS).

Oh!

Here is, as I think, the generally accepted definition of
convergence (Grauert, Lieb: D+I vol. 1,p. 41)

Konvergiert eine Folge (a_nu) gegen x_o, so nennt man x_o
Grenzwert
(Limes) der Folge ...
Eine nicht konvergente Folge heißt divergent.

(Grauert, Lieb: D+I vol. 1,p. 48): Mit Hilfe des
Grenzwertbegriffes
ist es möglich, in gewissen Fällen auch unendlich vielen
reellen
Zahlen eine wohlbestimmte Zahl als Summe zuzuordnen.

Obviously, in other cases, this is impossible, isn't it?

To what particular question do you reply to by this particular quote?
This particular quote you intent to support _which_ particular
statement?

I reply to KISS: "The value or sum or limit of a series" and to the
denial of the interpretation of the limit of the series as being the
sum of its terms, as one reads it in modern text books (of low
quality).

Reihe = series, Folge = sequence. Your quote is not about series at all.

The true story about series and sequences can be read, pardon wikipedia
again, here:

,−−−−[http://en.wikipedia.org/wiki/Series_%28mathematics%29#Formal_definition]
| Mathematicians usually study a series as a pair of sequences: the
| sequence of terms of the series: a0, a1, a2, ? and the sequence of
| partial sums S0, S1, S2, ?, where Sn = a0 + a1 + ? + an. The
| _notation_
|
| \sum_{n=0}^\infty a_n
|

| represents then a priori this pair of sequences, which is always well
| defined, but which may or may not converge. In the case of
| convergence, i.e., if the sequence of partial sums SN has a limit, the
| notation is also used to denote the limit of this sequence. To make a
| distinction between these two completely different objects (sequence
| vs. numerical value), one may sometimes omit the limits (atop and
| below the sum's symbol) in the former case, although it is usually
| clear from the context which one is meant.
`----

Would you also agree to Eulers result

1/(1−2) = 1 + 2 + 4 + 8 + ... = (−1) > 1? [(*)]

Euler's/Ramanujan's result actually is:

1 + 2 + 3 + ... = −1/12 (R)

So I would
like to see
your
calculation
step _by_
step
which leads
to your
statement
(*). Then I
can
demonstrate
your
error.

How would you accomplish
that?

By identifying your error.

By your usual unlogical and
wilful commenting?

No. By identifying your error as usual.

An error cannot be
demonstrated *by you* in
my statement,
because A) 1 + 2 + 4 + 8 +
... = (−1)
is the same nonsense as
1 + 2 + 3 + ... = −1/12.

I am going to identify the error in your
derivation of (*) after
you will have presented it.

B) The statement is not
mine but is a result of Euler.

The statement of Euler/Ramanujan is

1 + 2 + 3 + ... = −1/12 (R)

Your statement under discussion (*) is

1/(1−2) = 1 + 2 + 4 + 8 + ... = (−1) > 1? (*)

What now?

I would like to see _your_ calculation _step_
_by_ _step_ which
leads to your statement (*). Please do or
otherwise scrub (*).

If −− what I do not know −− Euler has
literally written (*) you
may name a reference or even better quote
his derivation.

I think you can find this in many books on history of math. I
recommend M. Cantor. (What I have stored in my memory
isn't always
labelled by a reference, nevertheless in most cases it is
correct.)

I would not rely on that.

But you would rely on the axiom of choice! LOL.

I meant that I would not rely on your memory.

Regards, WM

PS: I have found a reference: W. Mückenheim: Kleine Geschichte der
Mathematik − von den Anfängen bis ins 18. Jahrhundert (Skriptum
zur Vorlesung), Augsburg 2001, p. 110.

There are two issues which have aggregated now:

1. By writing

| Would you also agree to Eulers result
| 1/(1−2) = 1 + 2 + 4 + 8 + ... = (−1) > 1? [(*)]

you impute that Euler literally has written (*). An accepted proof that a person A has literally written B is _quoting_ literally from the work of A.

If I wrote a scientific article about this result of Euler's (which was also derived by Wallis, if I am right) then I would have studied his work. But I was only en passant mentioning an absurdity fitting well to another absurdity.

For the record: WM does not give quote for his claim so his claim that Euler has literally written (*) is not proved true (yet).

You did not provide us with such proof.

If Euler's result is wrong,

To begin with I doubt that (*) is of Euler's origin, i.e. that he literally has written (*).

you should be able to point that out.

"(−1) > 1" is mathematically wrong. That does not prove that (*) is by

Euler. I suppose it's a Mückenheim.

> If you accept Euler−Ramanujan, however, I do not see why you disagree
> with Euler alone.

I assume that (E)

| 1 + 2 + 3 + ... = −1/12 (R) [(E)]

is from Euler and/or Ramanujan. And I can't see any mathematically
logically valid path (manipulation) going from (E) to (*).

> 1 + 2 + 3 + ... = −1/12 and
> 1 + 2 + 4 + 8 + ... = (−1) fit very well together.

If you write them in two successive lines they are close together but
that does not constiute a derivation from (E) to (*).

> On the other hand we could conclude, if
> 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + ... = −1/12, then
> 1 + 2 + 4 + 8 + ... =< −1/12 − 3 − 5 − 6 − 7
> − ... =/= (−1)
> unless
> −1/12 − 3 − 5 − 6 − 7 − ... = (−1).

(*), which you attribute to Euler, is

1/(1−2) = 1 + 2 + 4 + 8 + ... = (−1) > 1? (*)

Please demonstrate only the first step of (A) = (E) on the way to (*).

I calculate as follows

(A, E)
<=> (1 + 2 + 4 + 8 + ...) = −1/12 − (3 + 5 + 6 + 7 + 9 + ...)

What the next step on your way to (*)?

> But why not? In principle, if Dik is right (as you seem to believe)
> then everything could be (−1).

Slow down a bit. It is late and I cannot follow your weird explanations.
You probably cannot redefine at will. But Dik simply defined what was

previously undefined.

> That would considerably simplify mathematics. We would no longer need computers! Students would always pass the exams. There would be no drop outs!

According to you we even don't need resource allocation conflicts when we accept WM's law of "least injustice". I will suggest you for Nobel Peace Prize.

> There remains only one problem, the last unsolved problem of mathematics: This mathematics cannot be applied for calculating Grocer bills and bank accounts. But that does not matter, "modern mathematics" finally has completely occupied the victory rostrum.

The grocers will ask you for calculating their bills. Perhaps the only kind of maths you'll master.

>> I do not judge whether your referenced own work literally quotes the work of Euler. I will surely not read it in order do unburden you from giving the appropriate quote _here_. You who claim that Euler has literally written (*) have to prove it _here_ (in sci.math).
>>
>> 2. To my best knowlegde Euler/Ramanujan are assumed to have written
>>
>> | 1 + 2 + 3 + ... = −1/12 (R) [(E)]
>>
>> and not (*). So to get to (*) <−> (E) you need some kind of formula manipulation which you have not presented at all. My claim is: Present the manipulation that yields (*) <−> (E) and I will show your error or retract my claim if I can't find an error.

> As far as I know, Euler simply applied the well known formula 1/(1−q) without paying attention to convergence,

That sounds less like Leonhard but like Wolfgang Euler.

> as was usual at those times
> (although Euler was the first to mention a convergence criterion.
> (You can find it on p. 10 of my "Die Mathematik des Unendlichen". Here it is off topic.) Leibniz and Jakob Bernoulli (1696) also agreed to
> 1/2 = 1/(1−(−1)) = 1 − 1 + 1 − 1 +−... = 0 + 0 + 0 + ... = 0.

Irrelevant even if the Pope agreed to it.

> With
> Dik's attitude of mathematics, we return to those times where the monk
> Grandi explained the creation of the world from nothing by just this
> effect: If you add enough zeros, then the result is 1/2.

irrelevant. evading the issue.

> Would not Dik's and your modern mathematics support just this position
> for actually infinitely many zeros?
>
> 0 + 0 + 0 + 0 + ... = 1/2

irrelevant. evading the issue.

> Of course. This must have been known to God already. There is no other
> way to create something from nothing. Or...???
>
> If we take an empty set, { }, {{ }}, ... Perhaps God created the world
> of empty sets?????

completely irrelevant. evading the issue.

>> I do not judge whether your referenced own work contains such
>> manipulation. I will surely not read it in order do unburden you from
>> giving the appropriate quote _here_ (in sci.math).
>>
>> To sum up: Until you perform your duties your claim Euler has written
>> (*) is vacuous.

> I will bear this verdict.

So we will bear that you retract "(*) is by Euler".

F. N.
--
xyz
.