Four color theorem: why this is not a proof and pointer to simple explanations

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A few weeks ago, I gave my 15 year old son a book on mathematical puzzles that included a description of the 4 color theorem. After spending some time drawing colored maps, he made a connection with another problem, that of the impossibility of connecting 3 houses with 3 different utilities (gas, water, electricity) without having two lines crossing.

In doing so, he came up with a "proof" for the 4 color theorem. I know it is not a proof, and I believe I know where it fails, but I can't explain it in a convincing way.

Here's the "proof"; I suspect that the problem is right with the first step.

1. If there is a map where 5 colors are needed, it must be because 5 different countries are touching each other.

2. Countries touching each other can be represented as (labeled) points linked by lines; two countries touching each other can be linked by a line which is not crossing any other line.

3. A map representing 4 countries touching each others can be drawn as follows

A −− B
| \ |
C −−D

with the addition of a line joining B to C going on the "outside" of the square ABDC.

4. A map representing 5 countries, some of them touching each other can be drawn as follows:

A −− B
| \ |
C D
| |

E /

An additional line can be drawn "inside" the ABDEC polygon joining A to E.
A line can be drawn on the "outside" joining B to C. A similar one can be drawn joining B to E.

The result is that all countries are touching each other _except_ D and C. However, no line can be drawn joining D and C without crossing an existing line (A to E "inside", or B to E "outside").

Hence, we cannot have 5 countries touching each other − and we don't need 5 different colors.

Any _simple_ explanation (or links) that I could pass along to my son would be greatly appreciated.

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