Re: must specify free variable for consistent operator on L^2(R)?

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- *From*: David C. Ullrich <ullrich@xxxxxxxxxxxxxxxx>
- *Date*: Fri, 15 Jun 2007 05:06:29 −0500

On Fri, 15 Jun 2007 02:05:53 −0000, Dan Greenhoe <dgreenhoe@xxxxxxxxx>
wrote:

> On Jun 13, 9:24 pm, David C. Ullrich <ullr...@xxxxxxxxxxxxxxxx> >
>
> > > [(Fx)(t)](w) = int_t x(t) e^{−iwt} dt
> >
> > No! That's wrong, even though you see things written
> > that way in various books (not in books written by
> > careful mathematicians). The point is that _functions_
> > have Fourier transforms, _numbers_ do not have Fourier
> > transforms. Here x is a function and x(t) is _not_ a
> > function, x(t) is a _number_ (namely the value of
> > the function x evaluated at t.) So that definition
> > should be written
> >
> > (Fx)(w) = etc.
> >
> > Here x is a function, Fx is another function, and
> > then (Fx)(w) is the value of that other function at w,
> > defined to be that integral.
>
> Got it −−− thanks ^__^
> And I think this kind of notation more easily adapts to operators on
> arbitrary vector spaces as in
> Fx
> where F is an operator F:X−−>Y on vector spaces X and Y
> and x \in X.
> Does that sound right or like someone pretending to be a real
> mathematician?

Um, yes.

Say we've defined the Fourier transform Fx of a function x as above. And now we want to talk about the Fourier transform of the function x(2t). You can't write

(Fx(2t))(w) = int x(2t) exp(−iwt) dt

Sad news :(

You could do this:
"If y(t) = x(2t) then (Fy)(w) = etc",

I try to avoid introducing new variables _if_ possible −−− too much clutter in math proofs sometimes.

Or you could invent a dilation operator D, or maybe D_2, defined by
(D_2x)(t) = x(2t).
(FD_2x)(w) = int x(2t) exp(−iwt) dt.

I like this one better (my humble opinion).

Here FD_2x must mean F(D_2x), since the other grouping, (FD_2)x makes no sense.

My understanding was that this is true by definition (of operator multiplication). That is, if A and B are operators and x is a vector then
(AB)x = A(Bx) by definition.

Right − I didn't know that we were familiar with that standard bit of notation.

Thank you again very much for your help and for spending so much time in providing that help. I greatly appreciate it.

Dan Greenhoe

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David C. Ullrich

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