

Re: Separation, Power and Countability.

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On Jun 19, 3:32 am, MoeBlee <jazzm...@xxxxxxxxxxxx> wrote:

On Jun 18, 9:01 pm, zuhair <zaljo...@xxxxxxxxxx> wrote:

I see it interesting to know that most of the members of Pw are undefinable, and what I mean by undefinable is any set x for which the following property hold:
 $\sim \text{EPAy}(y \text{ex} \leftrightarrow P)$, P here is called the defining formula.

You're just going to keep ignoring what I've said about that even though I've said it over a dozen times now, aren't you? I've told you over a dozen times that if you quantify 'P' in a second order formula then 'P' is a predicate variable and it's not a formula. What you keep posting over and over does not make sense in the form you are posting it.

A member (or an n-tuple of members) of a universe of a model for a language is definable in the language iff there is a defining formula, for the element, in the language.

I.e., the n-tuple of members of the universe is the unique n-tuple such that a formula in n free variables is satisfied, per the model, iff the variables are assigned to the respective coordinates in the n-tuple.

More simply, for just an individual member of the universe, it is definable iff there is a formula in one free variable such that that formula is satisfied iff its free variable is mapped to said individual.

But that is something we say ABOUT the language and its models and members and n-tuples of members of the universe of a model, but not IN the language itself. In the set theory itself we don't talk about definability, but rather we talk about definability in a meta-theory, in a meta-language, about the set theory in question.

If you overlook such considerations and instead think that

Re: Separation, Power and Countability.

'definability' is something that we're talking about in the theory itself, then you are bound to continual confusion. And you would not be in this confusion if you just read a couple of good books on logic.

I will call such sets 'indefinable sets' or simply 'indefinables'.

So these indefinables in Pw are all subsets of w of course, but these subsets doesn't have any formula that can define them. I wonder how the equinumerosity of these subsets with w is proved?

In the set theory itself we don't INDIVIDUALLY prove things about what is not defined. Rather, in your example, we prove UNIVERSAL GENERALIZATIONS about ANY infinite subset of w .

It is clear that all these indefinables in Pw are infinite subsets of w , but how can we prove that they are equinumerous to w .

We prove it as a GENERALIZATION; we don't prove it for each one individually.

Lets say that d is an indefinable subset of w .
Now we have $Ef(f:d \rightarrow w, f \text{ is injective})$, simply because $f(x)=x$ would do the job.
But how do we prove the opposite direction i.e.
 $Eg(g:w \rightarrow d, g \text{ is injective})$.

We prove it as GENERALIZATION. ' d ' there is variable upon which we will generalize, and ' d ' there does not stand for a certain specific set. So what you're asking reduces to the question of what is the proof that any infinite subset of w is denumerable. You will find such proofs in most textbooks of set theory.

The problem is that d is indefinable, so there is no rule by which we can have such an injection, perhaps I am confused here, since I think that the existence of an injection from w to any subset of w depends on definability of that subset using separation, i.e. if d is a subset of w , then we can find an injection from w to d if and only if d is definable.

Re: Separation, Power and Countability.

That's all a terrible confusion that is, as I mentioned, caused by your not understanding that 'definable' is not something we're talking about IN the language of the set theory itself.

I think I should be wrong. Since it appears somewhat clear that d even if it is undefinable still it appears that there should exist an injection from w to d , and by then d should be equinumerous to w .

So my question specifically is the following:
Question: Suppose d is an undefinable subset of w .
is there a proof of $\exists f(f:w \rightarrow d, f \text{ is injective})$?

Your question doesn't make sense. Saying ' d is undefinable' is something that we'd say in the meta-language. Saying 'For all d , if d is an infinite subset of w , then d is denumerable' is something we're saying in the object language, and there ' d ' is a variable that is universally generalized, without regard to any definability questions, since those questions couched in the in the theory itself.

There are people who know this subject much better than I do, so they may qualify what I've said with more precise and vastly more authoritative explanations. But to benefit from the generosity of such people you need first to learn the basics of the subject.

MoeBlee

What you said make sense. But it is not enough.

I asked for the proof of the following:
For any set d that is a member of Pw and is undefinable what is the prove of d being equinumerous to w ?

Your answer was: the question itself doesn't make sense.
because 'for any member of Pw ...such and such' is a sentence in the theory while the description ' d is definable' is in the metatheory..?
so the question doesn't make sense.

So what you want to say is that there is actually no question.

that was your answer in nutshell.

That's not enough.
The universal generalization you've mentioned is also no enough.

Re: Separation,Power and Countability.

Even is these sentences are as you mentioned, one in the theory and on in the metatheory, there should be a proof,
Perhaps this proof is not in the theory, Perhaps not in the metatheory, But there should be a proof somehow,somewhere. Otherwise this mean that the logic apparatus we have cannot answer such question.

Zuhair

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