Re: 3^n and primes

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- *From*: Robert Israel <israel@xxxxxxxxxxxxxxxxxxxxxxxxxxxxx>
- *Date*: Tue, 03 Jul 2007 00:39:34 −0500

quasi <quasi@xxxxxxxx> writes:

> On Mon, 02 Jul 2007 19:50:49 −0700, rer <reriker@xxxxxxxxx> wrote:
>
>> This is clearly not efficient, but seems curious. However, I am unable to explore it further because of rounding errors. I hope someone out there with a bigger number cruncher is curious, too, and will see if this just dies out quickly, or has something more too it.
>>
>> Consider 3^n where n is an integer greater than or equal to 2.
>>
>> Then let p+q = 3^n, such that p, q are consecutive integers

i.e. p=(3^n−1)/2 and q = (3^n+1)/2.

>> If 2n+1 evenly divides p or q, then 2n+1 is prime

> It fails occasionally.
>
> For n<1000, it fails for the following 5 values of n:
>
> n = 60, 351, 770, 864, 945

Let m = 2n+1. m is called a "weak probable prime to base 3" if m divides 3^(m−1) − 1 = (3^n − 1)(3^n + 1). In particular this happens if m divides p or q. So your numbers are probable primes, but not necessarily primes.

Actually m is an "Euler probable prime to base 3" if either m == 1 or 11 mod 12 and m divides p, or m == 5 or 7 mod 12 and m divides q.

Of the exceptions quasi found, the only one that doesn't correspond
to an Euler probable prime is n=770, m=1541, since 1541 == 5 mod 12
but divides 3^770 − 1 instead of 3^770 + 1.
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