

Re: limit of a sequence

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On Jul 7, 8:01 am, "Armando C." <xxxxxxx...@xxxxxxx> wrote:

I've a doubt about the following sequence limit, I tried many times to solve it, but I couldn't work it out.

$$\lim_{n \rightarrow +\infty} (1/(n^2+n+1) + 2/(n^2+n+2) + \dots + n/(n^2+n+n))$$

My book says result is 1/2, but frankly speaking I don't understand how it comes to it. If each term is infinitesimal how can sum be different from 0?

Thanks in advance.

The sum is bounded from below by:

$$\lim_{n \rightarrow \infty} \sum_{i=1..n} (i/(n^2+2n)) \text{ [why? I leave this to you].}$$

Using the formula for the sum of the numbers from 1 to n, it is easy to show that this limit is 1/2. So your given limit/sum is at least 1/2.

A similar line of reasoning will show you that it is also bounded from above by 1/2 (I'll leave it to you to figure out the details of this.)

By the Squeeze theorem, therefore, the limit is 1/2.

Note that, more generally, this is an explicit example that the sum of infinitesimals doesn't have to be 0.