

## Re: $3^n$ and primes

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*Source:* <http://sci.tech-archive.net/Archive/sci.math/2007-07/msg01077.html>

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- *From:* rer <reriker@xxxxxxxxxx>
  - *Date:* Sun, 08 Jul 2007 02:01:57 -0000
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On Jul 5, 10:21 pm, rer <reriker@xxxxxxxxxx> wrote:

On Jul 5, 12:02 am, quasi <q...@xxxxxxxxxx> wrote:

On Thu, 05 Jul 2007 01:53:14 -0000, rer <reriker@xxxxxxxxxx> wrote:

On Jul 2, 10:50 pm, rer <reriker@xxxxxxxxxx> wrote:

This is clearly not efficient, but seems curious. However, I am unable to explore it further because of rounding errors. I hope someone out there with a bigger number cruncher is curious, too, and will see if this just dies out quickly, or has something more to it.

Consider  $3^n$  where  $n$  is an integer greater than or equal to 2.

Then let  $p+q = 3^n$ , such that  $p, q$  are consecutive integers

If  $2n+1$  evenly divides  $p$  or  $q$ , then  $2n+1$  is prime

Someone contacted me directly, also. They noticed that when it fails,

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the remainder on the nonfailing part is 1 or  $2n$ . They suggested this did not seem to be coincidental.

It's not coincidental — it's an identity.

If  $2n+1$  divides  $p$ , then since  $q=p+1$ , it follows that  $q=1 \pmod{2n+1}$ .

If  $2n+1$  divides  $q$ , then since  $p=q-1$ , it follows that  $p=2n \pmod{2n+1}$ .

It doesn't matter whether or not  $2n+1$  is prime.

Thus, it's of no use in detecting false positives.

quasi

Thanks.

I had chosen  $3^n$  because it was the smallest to test, and I had hoped that it might work. Obviously, it alone is not sufficient. However, it was really just the first member of a series of "triples" of the form  $(a+b)(a-b)^{p-1}=c^p$ . I show some examples here: <http://mysite.verizon.net/~reriker/exppythag-primes.html>

Now, as you showed, when  $p=60$ , one of the "triples" for 3 is divisible by 121. However, someone contacted me directly, and they did a test for some of the other numbers when  $p=60$ . Minimally, the triples of 5 and 7 are \*not\* divisible by 121. So, on this larger test, 121 shows as not being prime.

Of course, it could fail at 351, etc. And even if it doesn't fail, it is impractical for small numbers, and may be impractical for large numbers due to the sizes. I don't think it would be necessary to test very many values for any particular value of  $p$ . But, I would be curious to see if this larger test also fails quickly or not. So, if anyone has the inclination, the hardware, and the spare cycles, when does this one fall over?

Thanks again :-)

Someone was nice enough to run the following Mathematica code that I turned into this file -- Warning! Warning! Warning! It's a relatively large file at 4+ Mb:

<http://mysite.verizon.net/reriker/10003n.htm>

Mathematica code:

```
Table[
Table[a = Floor[c^p/2] + 1; b = a - 1;
{p, 2p+1, c, Mod[a, 2p+1], Mod[b, 2p+1], Mod[c, 2p+1}},
{c, 3, 69, 2}
],
{p, 2, 1000}
]// MatrixForm
```

I have not had a chance to look at every  $p/2p+1$ . However, unlike when looking at just 3 for  $p < 1000$ , it does *not* show false positives at 60, 351, 770, 864, or 945.

Granted, it's still a relatively small sample, but it certainly looks very promising.

rer

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