

Re: General solution to equation

Source: <http://sci.tech-archive.net/Archive/sci.math/2007-07/msg01146.html>

- *From:* grove.matthew@xxxxxxxxxx
 - *Date:* Sun, 08 Jul 2007 09:01:24 -0700
-

On Jul 7, 7:43 pm, Raymond Manzoni <raym...@xxxxxxx> wrote:

grove.matt...@xxxxxxxxxx a écrit :

I need to solve the following equation for n, and I'm having a lot of trouble.

$$(s*(1+t)^n-s)*(1-z) = (s*(1+x)^n-(s+(s*d*(1-(1+x)^n)/-x)))*(1-c)+ (s*d*(1-(1+x)^n)/-x)$$

Any help finding a general solution for n would be much appreciated. It would even be helpful if someone could tell me that it's not possible to find a general solution so I stop banging my head against the wall. Thanks in advance.

The 's' factor appears everywhere and may be removed.

Let's note $L= 1-z$ and $R= (c-1) (1-d/x) + d/x$ then your equation is simply (if L not 0 that is z not 1) :

$$((1+t)^n - 1) = R/L ((1+x)^n - 1)$$

I'll define a function of n (with $r= R/L$) :

$$f(n)= ((1+t)^n - 1) - r ((1+x)^n - 1)$$

I'll suppose that t and x are different and not -1 and that you are searching a real positive solution for n

A simple solution of $f(n)= 0$ is of course $n=0$

Let's study the behavior of $f(n)$ for positive values of n.

The derivative of $f(n)$ is

$$f'(n)= \ln(1+t) (1+t)^n - r \ln(1+x) (1+x)^n$$

Re: General solution to equation

$f'(n) = 0$ iff $\ln(1+t)/\ln(1+x) = r \left(\frac{1+x}{1+t} \right)^n$
that is $n_0 = \frac{\ln(\ln(1+t)/\ln(1+x)/r)}{\ln((1+x)/(1+t))}$ verifies $f'(n_0) = 0$
 n_0 may be complex, real and negative or real and positive.

Since f is smooth, $f(0) = 0$ and since $\lim_{n \rightarrow +\infty} f(n) = \pm \infty$ you may obtain a positive solution for $f(n) = 0$ only in the last case (n_0 positive real).

In this case you may solve $f(n) = 0$ by iterations for example using Newton's method :

Start with a 'large enough' value of n (say $2 n_0$) and compute
 $g(n) = n - f(n)/f'(n) = n - \frac{((1+t)^n - 1) - r((1+x)^n - 1)}{\ln(1+t)(1+t)^n - r \ln(1+x)(1+x)^n}$
replace n by $g(n)$ and continue until required precision

Should this method be unstable for your t, x, r values then try one of the methods proposed here : http://en.wikipedia.org/wiki/Root-finding_algorithm

Hoping it helped,
Raymond

Thanks to both of you. This is very helpful.