Re: ** says: Definition: sum{i in N} i = 0

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- *From*: hagman <google@xxxxxxxxxxxxx>
- *Date*: Mon, 09 Jul 2007 10:00:05 −0700

On 9 Jul., 15:50, WM <mueck...@xxxxxxxxxxxxxxxxx> wrote:

> On 9 Jul., 15:05, hagman <goo...@xxxxxxxxxxxxx> wrote:
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> > On 9 Jul., 09:52, WM <mueck...@xxxxxxxxxxxxxxxxx> wrote:
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> > > On 9 Jul., 07:32, hagman <goo...@xxxxxxxxxxxxx> wrote:
> > >
> > > > On 8 Jul., 22:50, WM
> > > > <mueck...@xxxxxxxxxxxxxxxxx> wrote:
> > > >
> > > > > > On the contrary it is simple because Cantor himself considered his proof an existence proof for trancendental numbers.

Well, but that does not make transcendental number the *cause* for the uncountability of the reals. For example, the existence of transcendentals is also possible by approximation theory: sum 10^(−n!) is transcendental. Thus transcendental numbers exist. But this tiny fact doesn't prove uncountabilty of the reals.

Of course only the (asserted) existence of *all* transcendentals makes R uncountable. Those few transcendentals which were constructed by Liouville himself and the handful being found later on by Hermite, v. Lindemann, Schneider, Gelfand and others is certainly not sufficient to make anything uncountable. (By the way, it was Liouville's aim already to prove e transcendental.)

Let a_0 be a sequence of ones and twos.

You meant (a_n)?

Yep.

Then
sum a_n*10^(−n!)
is transcendental of Liouville type and the
set of these is
uncountable,
of course by the same diagonal argument as
used for the reals
themselves.

The set of all infinite sequences (a_n) of this kind is
uncountable.
For this sake you need not append any "*10(−n!)".

I need the factor 10^(−n!) in order to produce a convergent series
and indeed a *rapidly* converging series such that the limit
is transcendental and no two sequences (a_n) produce
the same limit.
Note that without the factor 10^(−1), sum a_n is not defined

But sequences are defined. And what I told you is
1) that you need not transcendental numbers in order to show that
there are uncountably many infinite sequences, and

Of course not. But my argument fits well to the contest it was in:
To show that there are uncountably many transcendentals without
showing that there are uncounatbly many reals but only counatbly
many algebaric numbers.
The map from 1−2−sequences to transcendentals described by
(a_n) |−> sum a_n*10^(−n!)

is an injective map from the obviously uncountable set of
1−2−sequences to the set of transcendentals about the cardinality
of which we want to make a statement.

2) that Liouville did not create uncountably many transcendental
numbers.

IIRC, he showed that any number that can be approximated
unusually well is transcendental and gave sum 10^(−n!)
as a simple example of an unusually well approximable number.
He would hardly have been surprised to hear that all the
numbers sum a_n*10^(−n!) are transcendental.

Regards, WM

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